PRINCIPLES OF NAVIGATION

by

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BHANDARKAR PUBLICATIONS

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Dedication

Dedicated to the memory of my wife, Kamala, whose constant inspiration and encouragement guided me in all my endeavours.

CONTENTS

PREFACE TO FIRST EDITION

This book is written consequent upon persistent demand from several students both past and present preparing for their certificates of competancy as Second Mates Foreign Going Examinations. The book covers the existing and the proposed new Indian M.O.T. syllabus for second Mates. The chapters are divided into various convenient topics in a simple and logical sequence so that it can be understood even by a weak student. Intricate explanations or too involved proofs are avoided. Many years of contact with the students coupled with several years of teaching experience in this specific subject have helped me immensely in presenting the subject in this lucid manner.

There are four test papers included at the end of the book. These papers follow the expected pattern for the new syllabus, wherein Principles of Navigation is a seperate two hour paper carrying 100 marks, instead of a combined paper with Practical Navigation under the existing syllabus.

The Indian syllabus differs from the the U.K. syllabus, to the extent that the principle of position fixing by hyperbolic and satellite navigational systems and bridge equipments are excluded from the purview of this paper. Indian examinations will therefore include a two hour paper carrying 100 marks on "Bridge Equipment and Watch Keeping", wherein practically, all bridge equipments are expected to be covered. That paper being a separate paper this book does not cater to this topic.

Practically all calculations on Nautical Astronomy are so designed as to exclude the use of the Nautical Almanac. If any data from the almanac is required to solve that question, the data is supplied in the question itself. By adopting this method, reprinting of several pages of Nautical Almanac for reference is avoided.

Though the book is primarily written to satisfy the needs of candidates preparing for their M.O.T. certificates as Second Mates, this book can also serve as an excellent reference book for those who are teaching or those who are in any way connected with the subject.

I am extremely thankful to Mr. K. R. Bhandarkar for his encouragement and guidance and for his consistent pressures on me to complete the book. Without this pressure, the writing of this book might have taken much longer. I am also thankful to the staff of Bhandarkar Publications who did much of the typing work and the artwork required. Above all, I am greatly indebted to my late wife, to whom this book is now dedicated, without whose encouragement and patience, this book would have remained a dream for me.

Though every effort is taken to avoid both typographical and clerical errors in calculations, I request all users of the book to kindly bring it to my notice if they do find any errors, so that we may rectify the same in subsequent editions.

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Bombay - 400 019.

Captain P. M. Sarma

PROPOSED SYLLABUS IN PRINCIPLES OF NAVIGATION - FOR M.O.T. EXAMINATIONS FOR SECOND MATES (F.G.)

In this paper, candidates may be asked to draw a figure reasonably to scale and to state the projection used.

(a) The shape of the earth. Poles, equator, meridians. Parallels of latitude. Position by latitude and longitude. Direction, bearing, distance, units of measurement. Difference of latitude, difference of longitude, departure mean and middle latitude, difference of meridional parts and the relationship between them.

Theory of great circle sailing. Calculation of initial and final course and the great circle distance. Small-circle on a sphere.

(b) The celestial sphere; definition on the celestial sphere, apparent motion on the celestial sphere. Declination. Azimuth, sidereal hour angle. The position of a body on the celestial sphere; azimuth with the altitude or declination with sidereal or local angle. The rising, culmination and setting of heavenly bodies. Circumpolar stars. Maximum azimuth.

(c) Solar system, earth moon system. Planetary motion. Earth's rotation and movement in orbit, eclipses, mean sun, ecliptic, first point of Aries, Equinox and Solistice. Sunrise, Sunset and twilight

(d) Time. Greenwich and other standard times, zone time, mean time, apparent time, sidereal time, equation of time, relationship between longitude and time. International Date Line.

(e) Local hour angle of heavenly body in time and arc. Greenwich hour angle of Sun, Moon, planets and Aries. Application of right angled and quadrantal spherical triangles.

(f) Correction of sextant altitudes including back altitudes; dip, refraction, horizontal parallax, parallax in altitude, semi-diameter and augmentation. Use of artificial horizon.

(g) Geographical position of a heavenly body. A circle of position and its practical application, i.e., position line. Intercept.

(h) Simple properties of mercator and gnomonic charts. Latitude and longitude scales, measurement of distance. Rhumb lines. Meridional parts.

PRINCIPLES OF NAVIGATION

CHAPTER I

THE EARTH

The Shape of the earth approximates to a sphere. It is flattened at the poles and bulged at the equator. The polar diameter about which the earth rotates is called its axis. The two points where this axis meets the surface of the earth are called the Poles of the earth; one called the North Pole and the other, the South Pole. The direction of rotation as viewed from space on the North pole is anti-clockwise.

The earth is an oblate spheroid. Mathematically this elliptical shape can be described by the amount by which it deviates from a circle. The figure of the earth is then represented as :-

$$\frac{\text{Equitorial Radius - Polar Radius.}}{\text{Equitorial Radius.}}$$

i.e. $$\frac{6378.16 - 6356.77}{6378.16} \quad \text{(values in Kilometers)}$$

$$= \frac{1}{298.25}$$

This fraction is called the "Compression" of the terrestrial spheroid.

Definitions (see fig. 1)

Great Circle is a circle on a sphere whose plane passes through the centre of the sphere. Thus if one cuts a sphere along a G.C., the plane will pass through the centre and divide the sphere into two equal halves

Properties of a G.C. (1) It divides the sphere into two equal halves. (2) It is the shortest distance between any two points on a sphere. (3) One and only one G.C. can be drawn between two points on a sphere unless, the two points are diamerically opposite in which case, infinite number of G.Cs can be drawn through these two points. Such diametrically opposite points are called the "Poles" of that particular G.C. and these poles are 90° away from any point on that G.C.

Examples of G.C. on the earth are : (1) Equator (2) All meridians.

Small Circles : Any circle on a sphere whose plane does not pass through the centre of the sphere is called a small circle.

Examples of small circles on the earth are parallels of latitude.

The Co-ordinates used to indicate any point on the earth are called the Latitude and Longtitude.

Equator is a G.C. on the earths surface running midway between the poles and is the reference circle from which latitude are measured North or South. It divides the earth into two equal halves, North & South hemispheres.

Poles of the earth are the two points 90° away from any point on the equatior and are the points through which the axis of the earth is assumed to pass. All meridians on the earth pass through both the poles.

Parallels of Latitude are small circles on the earth running parallel to and Northward or Southward of the equator.

Meridians are half the arc of Great Circles running from Pole to Pole and meeting the equator and parallels of latitude at right angle.

Prime Meridian is that specific Meridian through Greenwich from which longitudes are measured east or west. This is the merdian of zero degrees longitude.

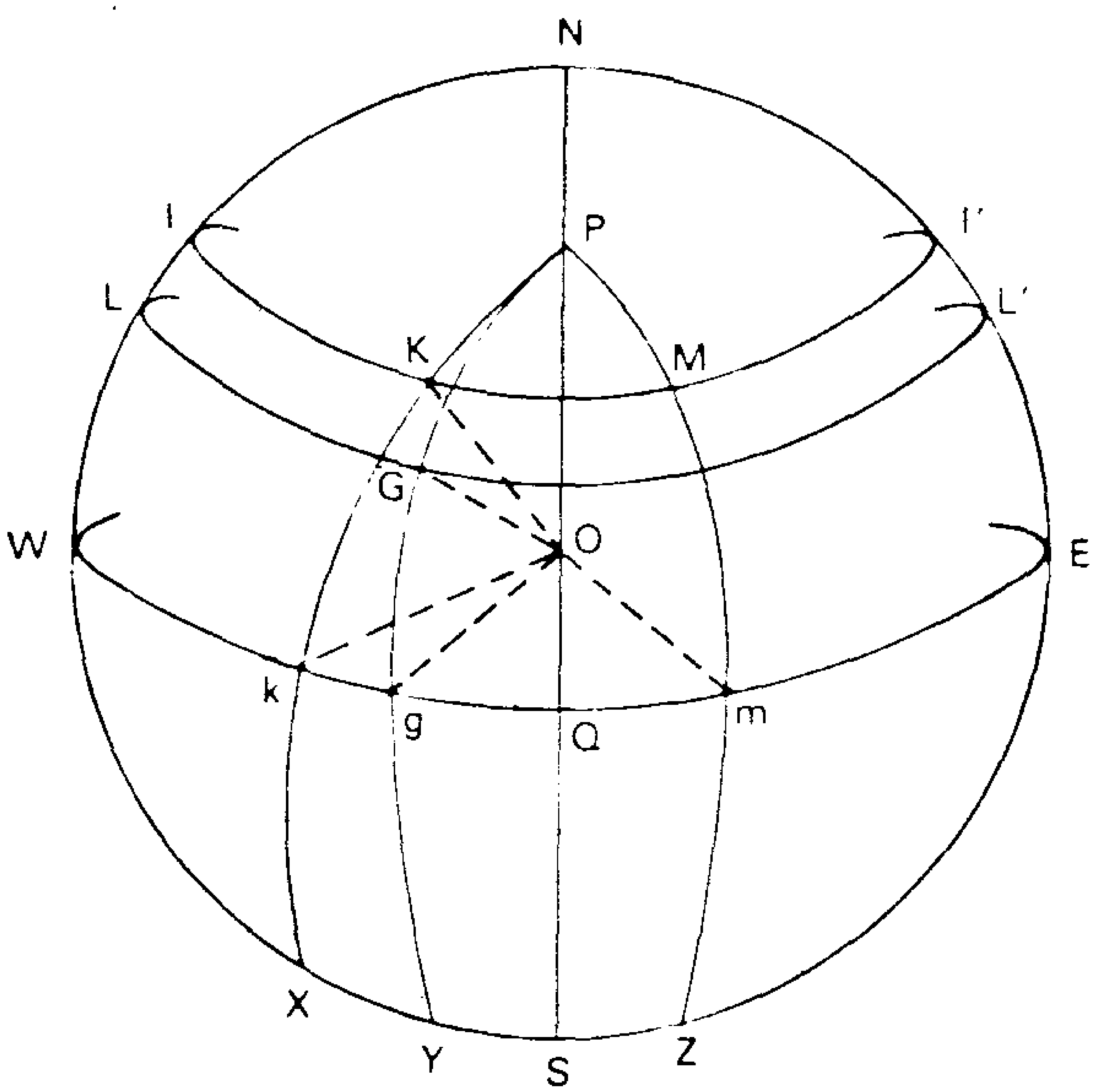


Fig. 1

The Earth.

P	Reps	North Pole
WQE	Reps	The Equator (Zero degree lat)
LL'	Reps	Parallel of Lat of 'G'
II'	Reps	Parallel of Lat of 'K'
O	Reps	Center of the Earth.
G	Reps	Greenwich.
PGgY	Reps	Meridian of G (Prime Meridian)
K & M	Reps	Two places on Latitude II'
PK k X	Reps	Meridian of K.
PM m Z	Reps	Meridian of M.
arc g G or $G\hat{O}g$	Reps	Lat of G.
arc kK or $K\hat{O}k$	Reps	Lat of K.
arc kg or $k\hat{P}g$.	Reps	Long. of k.
arc g m or $g\hat{P}m$	Reps	Long of M.

arc L1 or L'1' or $K\hat{O}G$ reps D' Lat between G & k

arc km or $k\hat{P}m$ or $k\hat{O}m$ "D'long between K & M.

arc Km Reps. Departure between K & M.

Latitude of a place is the arc of the meridian or the angle at the centre of earth on the plane of the meridian contained between the equator and the parallel of latitude passing through that place and is expressed North or South of the equator.

Longitude of a place is the shorter arc of the equator or the angle at the pole or the angle at centre of the earth on the plane of the equator contained between the prime meridian and the meridian passing through that place and expressed East or West.

Difference of Latitude (D'lat) is the arc of the meridian or the angle at the centre of the earth on the plane of the meridian between the parallels of Latitude of the two places; and named North (N) or South (S) depending on the direction travelled.

Difference of Longitude (D'Long.) is the shorter arc of the equator or the angles at the pole or the angle at the centre of the earth on the plane of the equator, contained between the meridians of the two places and qualified East (E) or West (W) depending on the direction of travel.

Departure The East-West distance between any two given meridians measured along a parallel of latitude and expressed in nautical miles is called departure and is qualified E or W depending on the direction of travel..

As the meridians on the earth's surface converge towards the poles the departure between two given meridians will continously decrease with increasing Latitude. At the equator, the departure will be equal to the D'Long and at the poles the departure will be zero. The departure & D'Long are mathematically connected by the equation :

$$\frac{\text{Departure}}{\text{D'Long}} = \text{Cos. Latitude.}$$

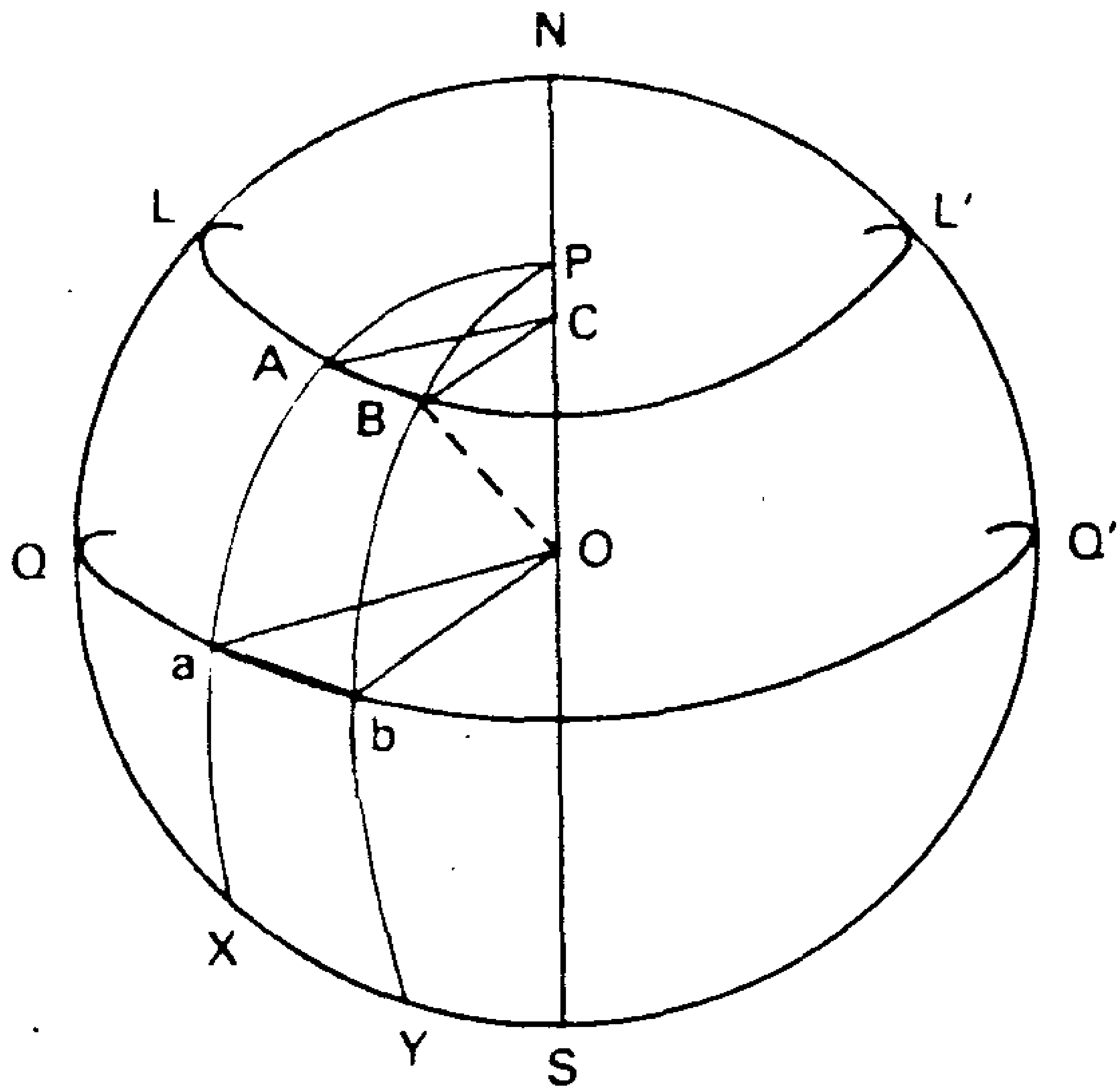


Fig. 2

NS reps axis of earth
A & B reps two places in Lat LL'
QQ reps equator
PA aX reps. long of A
P BbY reps. long of B
AB reps. departure
ab reps. corresponding D'Long
O reps Centre of earth
C reps. Centre of Small circle LL'

Proof

$$\frac{\text{Dep.}}{\text{D'long}} = \frac{AB}{ab}$$

Note : ABC & a bO are two $\triangle^s$ on parallel planes with points C & O lying on the axis. Angles at C & O are each 90° which these $\triangle^s$ make at the axis. Hence $\triangle^s$ ABC & abO are similar ($\because$ arcs are Proportional to the radii.)

$$\therefore \frac{AB}{ab} = \frac{AC}{aO} = \frac{BC}{bO}$$

Construction : Join BO

Since bO = BO (both radii of same meridian)

$$\frac{BC}{bO} = \frac{BC}{BO}$$

In $\triangle$ BCO $\because$ $\hat{C} = 90°$

$$\frac{BC}{BO} = \text{Sin } B\hat{O}C \text{ or Cos. } b\hat{o}B$$

But $b\hat{O}B$ = Lat of B

Hence $\frac{Dep}{D'long}$ = Cos Lat (Proved)

In proving the above, two assumptions have been made :

(1) that the places A & B are on the same parallel of Lat

(2) that the earth is a true sphere.

If the two places A & B are in two different Latitudes, as often happens, the formula is modified to read,

$\frac{Dep}{D'long}$ = Cos Mean Latitude, provided that places A & B are reasonably close together on the earth, i.e. approximately not more than about 200 miles apart. This is because, within this short distance, the curvature of the surface of the earth as a sphere and as a spheroid does not differ to any appreciable extent. For greater distances however, this approximation is not true and hence the forumula is further modified to read as :

$$\frac{Dep}{D'Long} = \text{Cos Mid-latitude.}$$

√ **Middle lat.** is defined as that latitude in which the relation ship Dep/D'long = Cos midlat truly holds good. One way of finding this middle lat is to apply a small supplementary correction to the mean Lat. This correction which is sometimes additive to or sometime subtracted from the Mean Lat to obtain the middle lat. is given in the older edition of Norie's Nautical Tables. In the later editions of Norie's this correction table is omitted because mid-lat can also be found from the formula

$$\frac{D'\ Lat}{DMP} = \text{Cos Mid Lat.}$$

As will be evident from the principles of Mercator's sailing $\frac{Dep}{D'Long}$ bears the same ratio as $\frac{D'Lat}{DMP.}$ (See figure 3).

)MP is the abbreviated form for Difference in Meridional parts

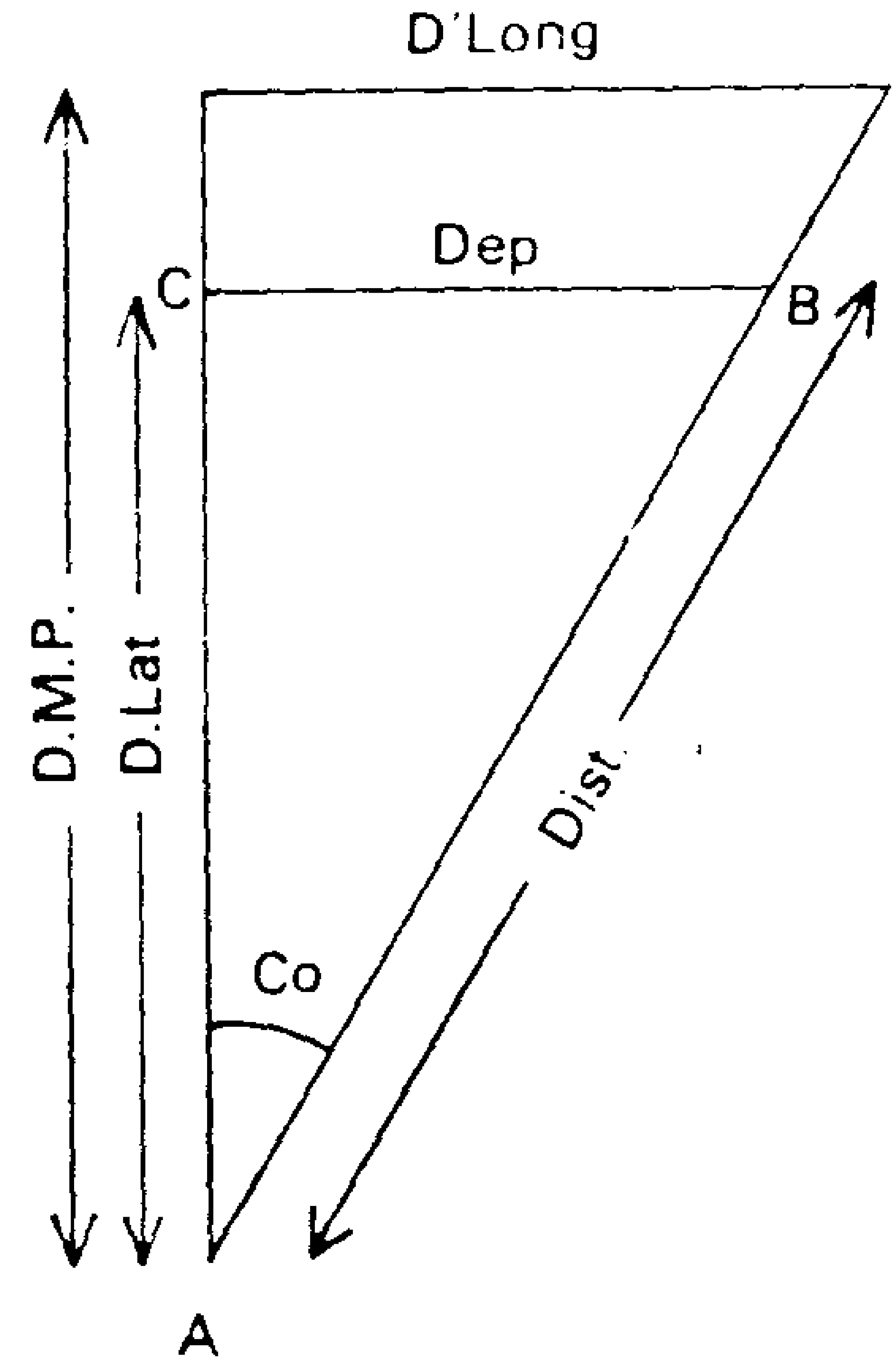


Fig. 3

/**leridional parts** are defined as the Latitude Scale of a mercator's hart, expressed in terms of its Longitude Scale. i.e. the number f times one mile of longitude will fall between the equator and a iven latitude on a Mercator's chart is the M.P. for that Latitude. his quantity is tabulated for each minute of Latitude in the leridional parts tables in Nories or other standard nautical bles. These tables take into account the actual compression of e earth as discussed earlier.

eocentric Latitude is the angle at the centre of the earth tween the plane of the equator and the line joining the server to the centre (OĈE in fig 4.).

eographical Latitude is the Latitude of position observed. In fig et O be the observer and K be the centre of curvature of the eridian at O

) is therefore the radius of curvature of the meridian and will et the circumference at right angles at O and hence this is the

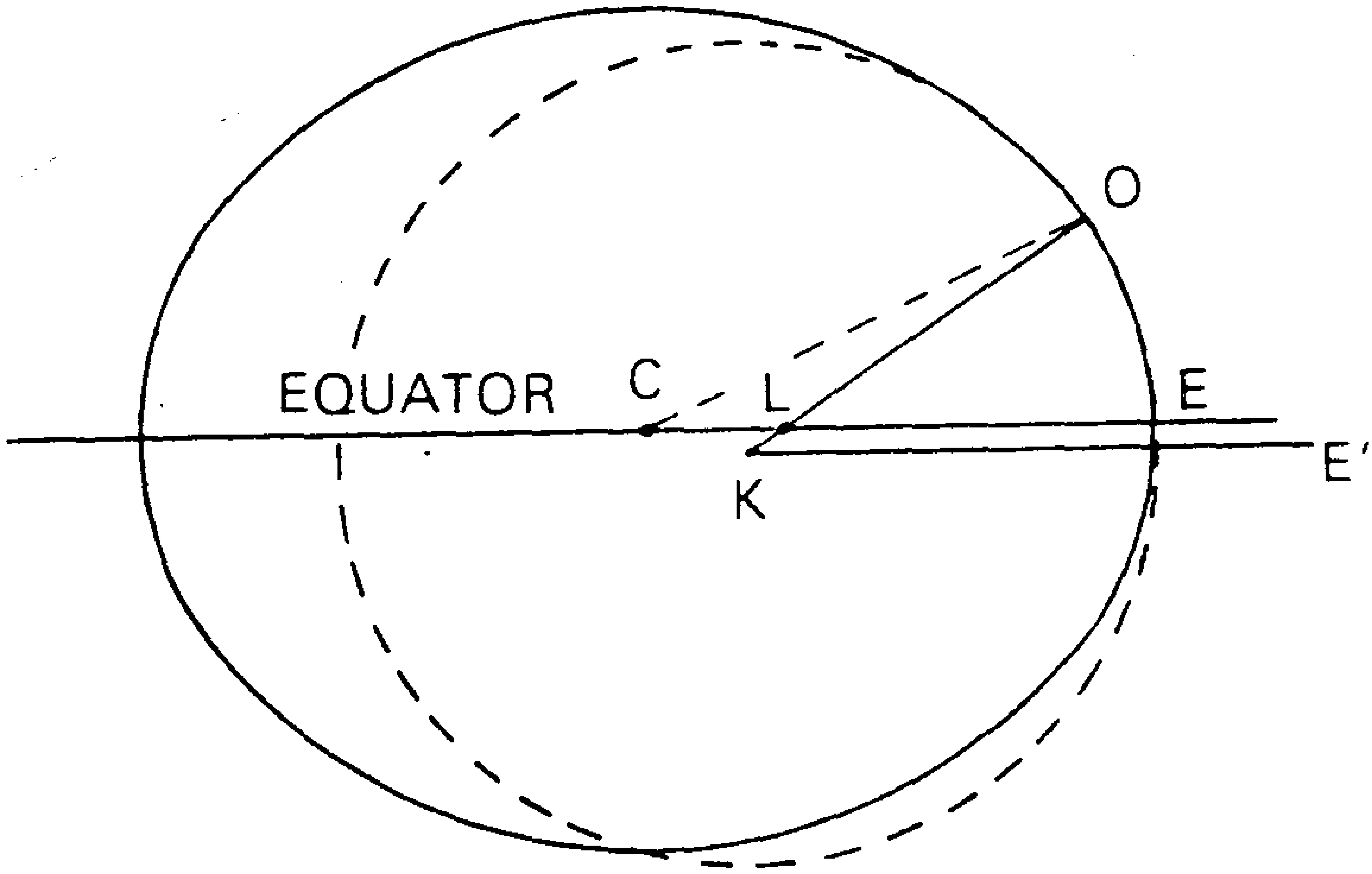


Fig. 4

vertical at O. The Geographical lat. is the $O\hat{K}E'$ which is equal to $O\hat{L}E$. Geographical lat. of an observer is therefore the angle between his vertical and the plane of the equator. The earth being an oblate spheroid, the radius of curvature of this elliptical shape will be greater at the poles and least at the equator.

In navigation, the term Latitude means Geographical Lat. At the equator and at the poles there is no difference between Geocentric and Geographical latitude. Maximum difference of about 11' of arc occurs in lat. 45° North or South.

The Geocentric Lat. can be expressed by the approximate formula : Geocentric Lat = $\varnothing - 11.6' \sin 2\varnothing$

Where $\varnothing$ is the Geographical Latitude.

Measurement of Distance. In navigation, the unit for measurement of distance is called a **Nautical Mile.** This is defined as the length of the arc of a meridian subtending an angle of one minute at the centre of curvature of the meridian. In otherwords it is the length of one minute of Geographical Latitude. As seen in the earlier section, since radius of curvature is longest at the poles and shortest at the Equator and since lenth of arcs are proportional to the radii, the length of the Nautical mile is maximum at the poles and least at the equator. At the poles its length is 6108 ft. (1861.7 m) and at the equator it is 6046 ft. (1842.9 m). In practice, an arbitrary length equal to 6080 ft. (1853 m) is considered to be the length of a nautical mile.

The actual length of a nautical mile in metres, in any latitude is obtained from the formula.

n.m = 1852.3 - 9.4 Cos 2 Ø

Where Ø is the latitude.

Geographical Mile is the length of one minute of arc of the equator or the length of one minute of D' long on the equator. This has a constant value of 1855.3 m (6087 ft).

Statute Mile is an arbitrary length equal to 5280 feet. There is no scientific basis on which this is arrived at.

Knot is the unit of speed and is meant to convey that the given speed is so many nautical miles per hour.

Exercise 1.

I. Find D' Lat. and D' Long. between the following positions.

	From (A)		To (B)	
1.	30° 18' N	10° 15' W	39° 32' N	20° 25' W
2.	23° 52' N	17° 27' W	32° 15' N	5° 16' W
3.	19° 30' N	50° 32' E	8° 16' N	73° 10' E
4.	13° 54' N	120° 14' E	2° 10' N	101° 09' E
5.	35° 22' S	9° 18' E	14° 19' S	30° 12' W
6.	03° 19' N	178° 30' E	8° 25' S	167° 18' W
7.	14° 53' S	112° 25' E	5° 15' E	80° 30' E
8.	12° 17' N	02° 42' E	2° 48' S	9° 30' W
9.	22° 12' N	8° 16' W	31° 19' N	7° 32' E
10.	60° 23' S	150° 35' W	20° 18' S	110° 33' W

II. Prove $\frac{\text{Dep.}}{\text{D' Long.}}$ = Cos Lat.

III. **Convert the following Dep. into D' Long.**

	Dep	**in**	**Lat.**
1	420' W	in	30° 40' S
2	300' E	in	10° 18' N
3	200' E	in	00° 54' N

IV. **Convert the following D' Long. into Dep. in the given latitude :**

	D' Long.	in	Lat.
1.	3° 20' W	in	18° 50' N
2.	31° 40' E	in	7° 18' S
3.	5° 30' W	in	50° 22' N

V. At what rate will an observer in Lat. 60° N be carried round the earth axis ?

VI. Why is the length of a nautical mile at the poles greater than at the equator.

VII. Find the length of a nautical mile in meters in lat.30° 00' S

VIII. Find the Geocentric Latitude corresponding to the Geographical Latitude at of 30°.

Answers

		D' Lat.		D' Long.	
I.	1.	9°	14' N	10°	10' W
	2.	8°	23' N	12°	11' E
	3.	11°	14' S	22°	38' E
	4.	11°	44' S	19°	05' W
	5.	21°	03' N	39°	30' W
	6.	11°	44' S	14°	12' E
	7.	20°	08' N	31°	55' W
	8.	15°	05' S	12°	12' W
	9.	9°	07' N	15°	48' E
	10.	40°	05' N	40°	02' E

III. (1) 488.3' (2) 304.8' (3) 200.0'

IV. (1) 189.25 (2) 1887.2 (3) 210.5

V. 450 m.p.h.

VII. 1847.6 m

VIII. Geocentric Lat = 29° 49.96'

CHAPTER II

NAVIGATIONAL CHARTS

(A) MERCATOR'S CHART

A Chart is a representation of the curved surface of the earth on a flat surface. Whenever this is attempted a certain amount of distortion is unavoidable, similar to what would happen if a semi-globular orange peel is flattened out on a table.

The form the distortion would take depends on the type of projection used. As long as the distortion inhernt in a particular projection has only a minimum effect upon the use to which the chart is put to, then such a projection could be selected for that purpose. The projection selected for Navigational charts is the Mercator's projection. This is sometimes referred to as a 'Cylindrical' projection.

Principle : Imagine a transparent globe with all latitudes and longitudes marked in dark ink and with a source of light placed in the centre. Imagine also a paper rolled in Cylinder fashion tangential to the equatorial plane of the globe. Now observe the shadows cast by the parallels of latitudes and longitudes on the paper. If this paper was a photographic paper this would print these shadows on the paper and when the paper is unrolled, we have the Mercator's chart. (See figure 5).

It will be observed here that all meridians appear as equidistant parallel lines and all parallels of latitudes also appear as parallel lines at right angle to the meridians, but at distances further and further away as we approach the poles.

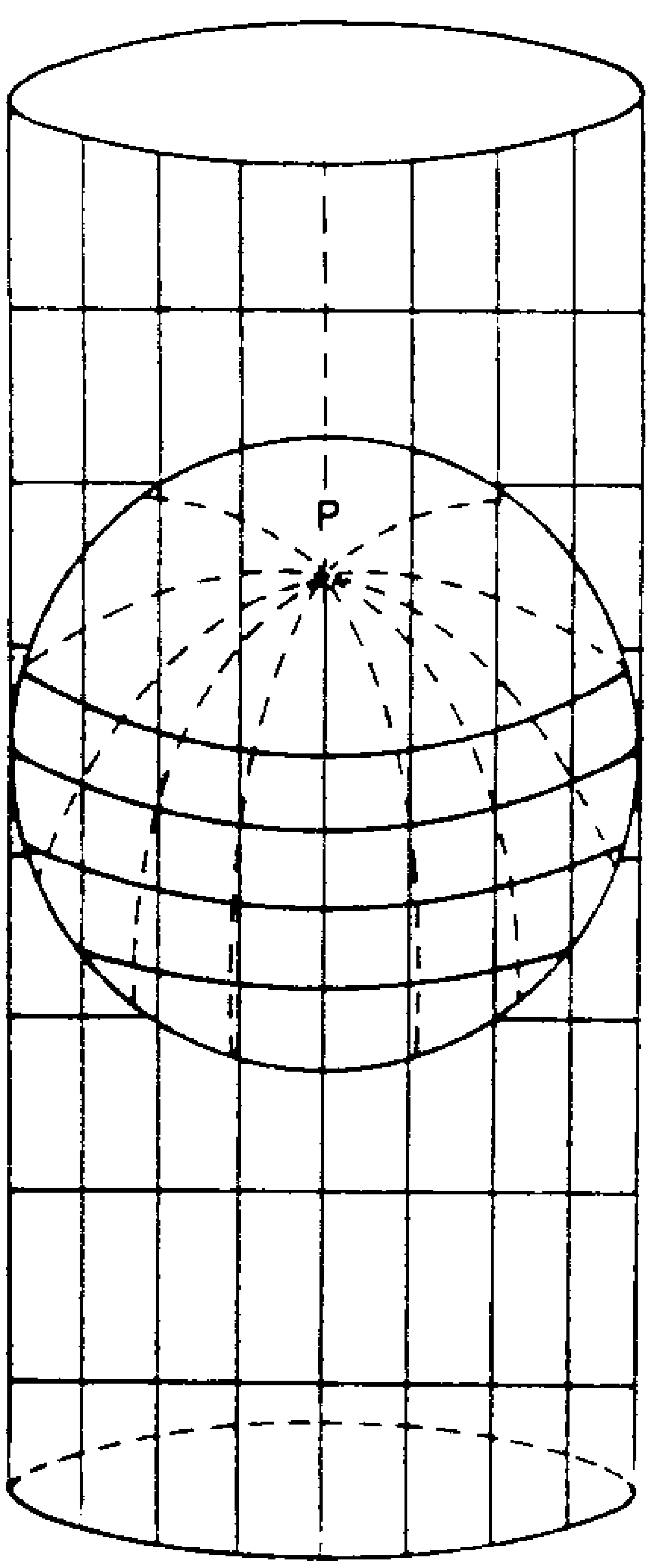


Fig. 5

The very fact that all meridians are shown as parallel lines, it becomes obvious that the departure in any latitude has been automatically stretched out till it becomes D'long.

Since D'long = Dep x Sec Lat; the E/W distance on the chart has been increased by the sec. of lat. in which it occurs on the earth. In order that all angles and distances can be shown correctly in a given area, if E/W distances are increased by Sec. of lat. it is obvious that the N/S distance also have to be increased by the same ratio. In fact that is exactly what is done. North/South measurements being latitudes on the chart, latitude scale is increased by the Secant of latitude. Distance apart between equal D'Longs. being constant, the longitude scale of Mercator's chart is constant.

Construction of Mercator's chart

Suppose we have to construct a Mercator's chart of an area between 20° & 23° N and longitudes 60° & 64° E. First we select a longitude scale, suitable for the size of the chart paper we have. This scale will be constant.

There are three methods by which this area could be represented on a Mercator's chart.

Method I : Graphical :- Draw a base line to represent the lowest latitude of 20° North (fig 6). Erect five perpendiculars on this line at 15 cms apart commencing from the left hand margin, the first one representing 60° E longitude and the rest will represent 61° to 64° E in their serial order. At 60° longitude, on the base line, make an angle equal to 20½° which is the mean between 20° & 21° lat. The length of hypotenuse of this right angled triangle so formed will represent the distance equal to the secant of the angle 20½° corresponding to the base of 15 cms. Mark off this length from the base line on each Meridian and join them. The line so obtained will represent the latitude of 21° N.

On this latitude, now make an angle equal to 21½° and cut off on each meridian again, a length equal to the new hypotenuse and join them to give the lat. of 22° N.

Repeat the process again on lat. 22° by making a angle equal to 22½° to obtain the lat. of 23° N & so on to obtain various latitudes even over 23° N, if required. We now have a mercator's chart of the area contemplated. It can be appreciated that, an

Let us say this scale is 15 cm = 1° of Longitude

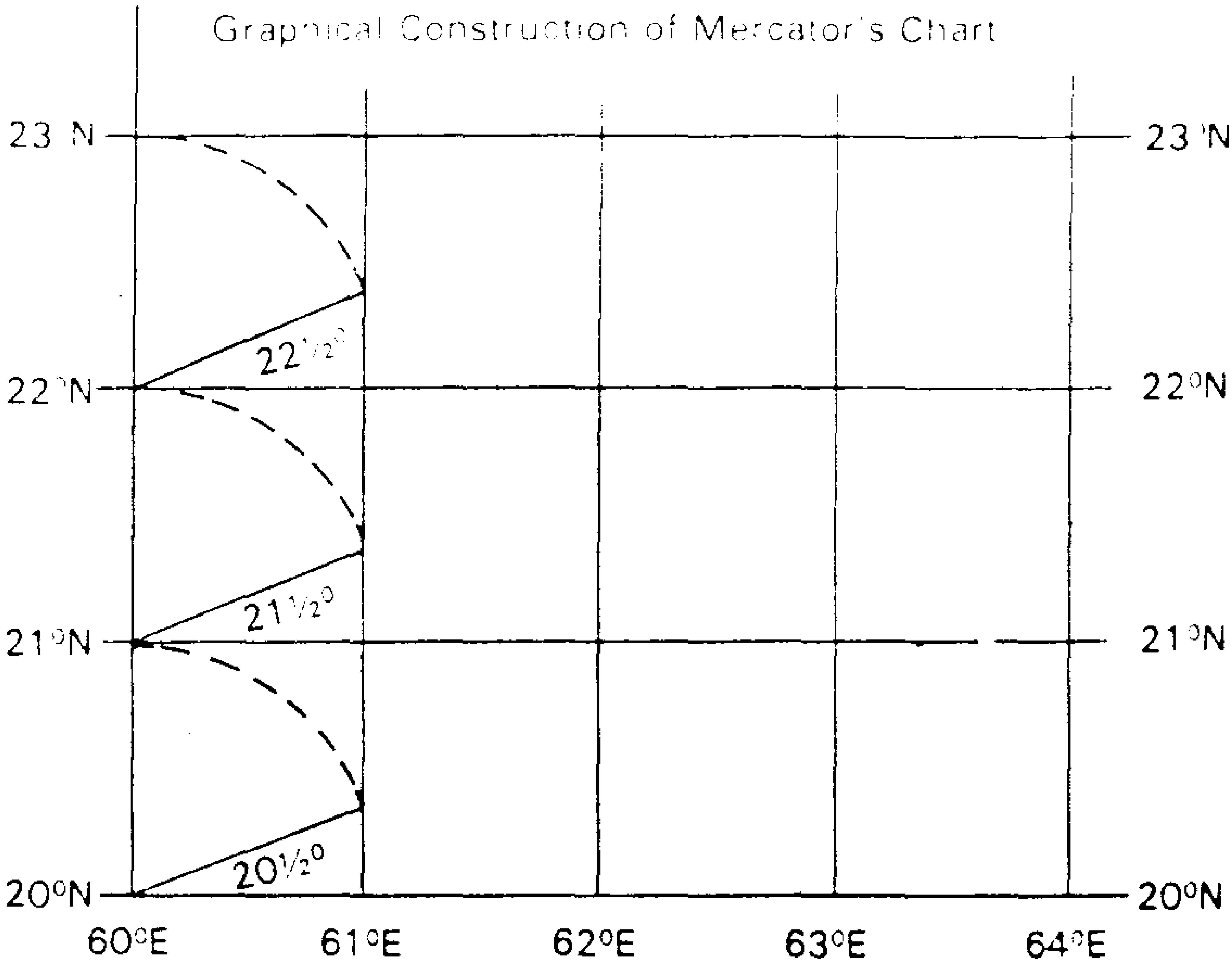


Fig. 6

angle of 20½ or 21½ or 22½ etc. cannot be drawn very accurately and hence the chart obtained, though it is a mercator's chart, it is not extremly accurate.

Method II. In this method, though the process of drawing the base line and erecting the meridians etc. are the same as the graphical method explained above, instead of making an angle and measuring off the length of the hypotenuse, we calculate this length. Since the length of the hypotenuse is in fact the distance between given two latitudes corresponding to the constant long scale, we can say :-

Lat. scale = Longitude scale × Sec Mean latitude.

Lat. scale between 20° & 21° N = 15cm × Sec 20½° = 16.014 cm.

No.		Log.
15	=	1.17609
Sec 20° 30°	=	0.02841
16.014	=	1.20450

The length 16.014 cm is marked off all meridians from the base line of 20° N. Join these points to obtain lat. of 21° N. Work out again the distance between 21° & 22° using the above formula :- Lat. scale = 15 × sec 21½, and draw the latitude of 22° North. Similarly for lat. of 23° N, use Lat. Scale = 15 × sec 22½° & so on. Even the chart so produced is not very accurate because, the use of mean latitude, though accurate enough for practical purposes, has its inherent drawback as it assumes the earth to be a true sphere, whereas in actual fact the earth is a spheroid. This assumption is overcome by using Method III.

Method III. Having selected the Long Scale, as before draw the base line representing 20° N lat. and erect perpendiculars as before to represent the required longitudes.

From the very definitions of the meridional parts (pp 7) it is evident that the length of one minute of Meridional part is equal to the length of one minute of Longitude. We utilise this property of M P to obtain the latitude scale of the chart as follows :-

M P for 20° N	=	1217.14
M P for 21° N	=	1280.81
D M P	=	63.67

The scale for our chart being 15 cm = 1° of Long.
We can say 60' of long. = 15 cm

$$63.67' \text{ D.M.P} = \frac{15 \times 63.67}{60} = 15.918 \text{ cm (Say } 15.92 \text{ cm).}$$

We mark off this distance 15.92 cm on each meridian from the lat. of 20° N and join these points to provide us the lat. of 21° N. Similarly by using DMP between 21° & 22°, lat of 22° N can be drawn and DMP between 22° & 23° will provide the 23° N lat & so on.

The fact the M.P. tables allow for the compression of the earth, this method of construction is very accurate in comparison to the first two methods described. Thus we get the relationship

$$\text{Lat Scale} = \frac{\text{Long. Scale} \times \text{D M P}}{\text{No : of minutes of Longitude in the Scale of longitude.}}$$

Rhumb Line is a line which cuts all meridians at same angle. On the earth's surface, this will be a curved line spiralling towards the poles, but never actually reaching the poles. (See fig. 7)

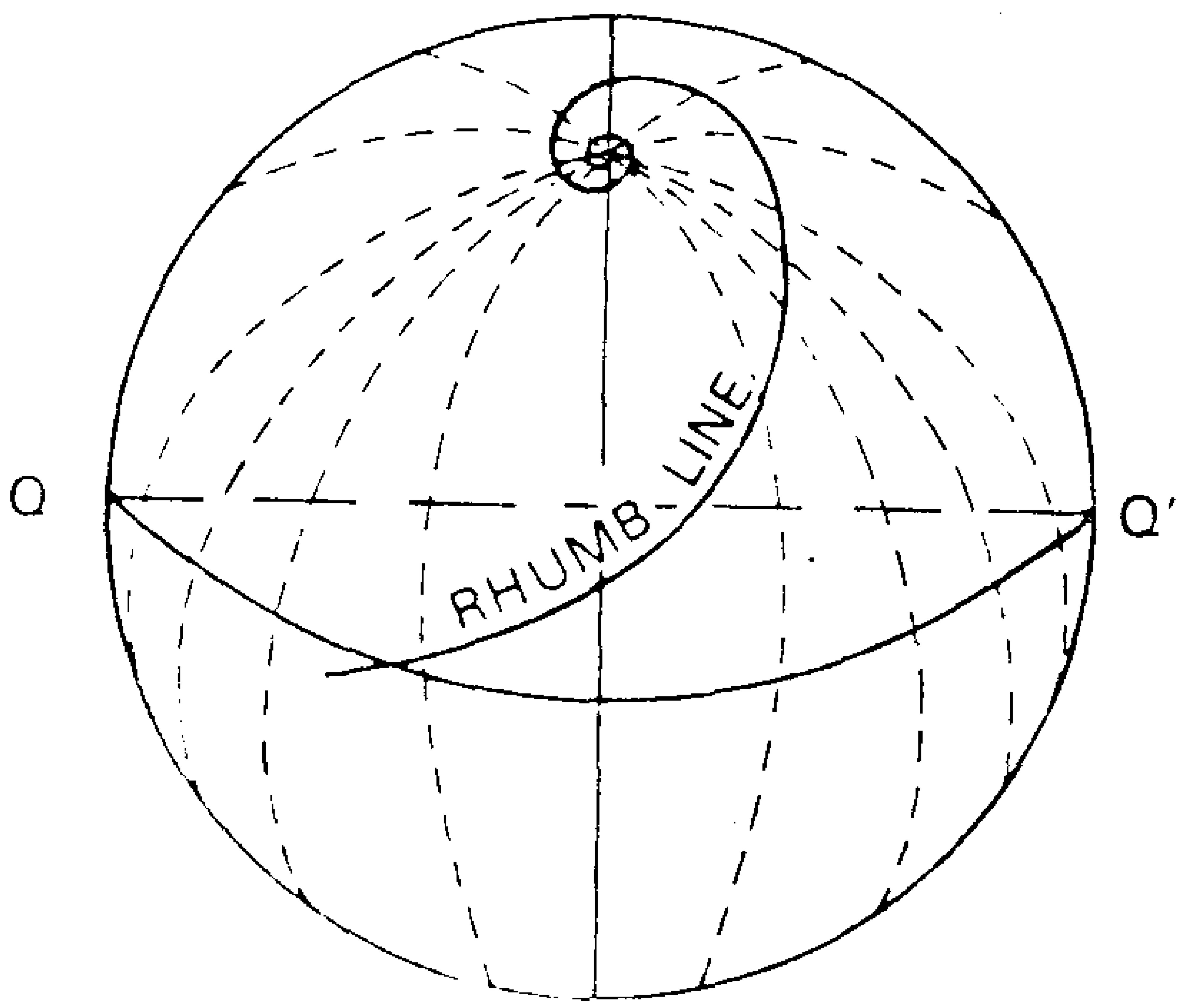


Fig. 7

On a mercator's chart however, all meridians, being parallel straight lines, a rhumb line will also be a straight line, thus cutting all meridians at the same angle.

Advantages of a Mercator's chart :

(1) Rhumb Lines can be represented as straight lines.

(2) Courses and bearings can be readily drawn and can be transferred from one part of the chart to the other without any loss of accurcy.

(3) Distances can be readily measured as the scale of latitude is also the scale of distances.

(4) All positions on the chart are correctly represented in their relative positions as they appear on the earth.

Disadvantages :

(1) Great circle courses cannot be readily drawn.

(2) Mercator's charts of the polar regions cannot be produced, as the distortion becomes execessive.

(3) Areas of land masses appear to be deceptive. For example on two charts, of equal scale, a small island like Greenland situated in a high latitude appears as big as the continent of Africa, where the equator passes almost through its middle. This is due to considerable distortion affecting the chart in high latitudes.

Of these disadvantages only the first viz. the inability to represent a G.C. course readily on a Mercator's chart, can be considered as somewhat relevant to the practice of Navigation. The other two disadvantages are not of any consequence.

Natural scale of a chart or a map is the ratio between one unit of length on the chart to the actual length on the earth's surface expressed in the same units in a specified latitude.

Example : A natural scale of $\dfrac{1}{50,000}$ in lat 40° N will imply that one cm on the chart will be equivalent to 50,000 cms on the earth in latitude 40° N. On every Admiralty chart, a natural scale is given under the title.

Given the long Scale of a mercator's chart in cm the natural scale in any latitude can be calculated from the formula. :-

$$\text{Nat. scale} = \frac{1' \text{ of Long Scale} \times \text{Sec Lat}}{100\ (1852.3 - 9.4\ (\cos 2\ \text{lat})^*}$$

* Note : Cosine values of angles exceeding 90° are negative

Example :- Find the Natural scale of a mercator's chart in lat 40°, given that the long scale is 12 cm to one degree of longitude.

$$\text{Nat Scale} = \frac{12 \times \sec 40^\circ}{100\,(1852.3 - 9.4 \cos 80^\circ)} = \frac{0.26108}{185066.77}$$

$$= \frac{1}{708850.8} \quad \text{say} \quad \frac{1}{708851}$$

(B) GNOMONIC CHART

One of the material disadvantages we found with Mercator's projection was the inability to represent great circles readily. This difficulty is overcome by using a Gnomonic chart. As the name implies, this chart is produced on the Gnomonic projection which is sometimes called the Tangential projection.

Principle of Gnomonic Projection : Imagine a transparent globe with all latitudes and meridians marked in dark ink with a light source at the centre. Hold a sheet of paper flat and tangential to the surface of the globe, at one of the poles and observe the shadows of the parallels of lat & meridians on this sheet of paper. What you have, is a Gnomonic chart. Pole being the tangent point, this chart is the polar Gnomonic chart. If the paper is held tangential to the equator, the resulting projection is termed Equitorial Gnomonic. If any other lat is the tangent point, then it is called an oblique or "Middle Latitude" Gnomonic at that specified lat. Simplest of these is Polar Gnomonic.

Polar Gnomonic : Most charts used for Navigation, particularly in higher latitudes are projections on Polar Gnomonic. It is easy to visualise how the projection would, appear. All meridians

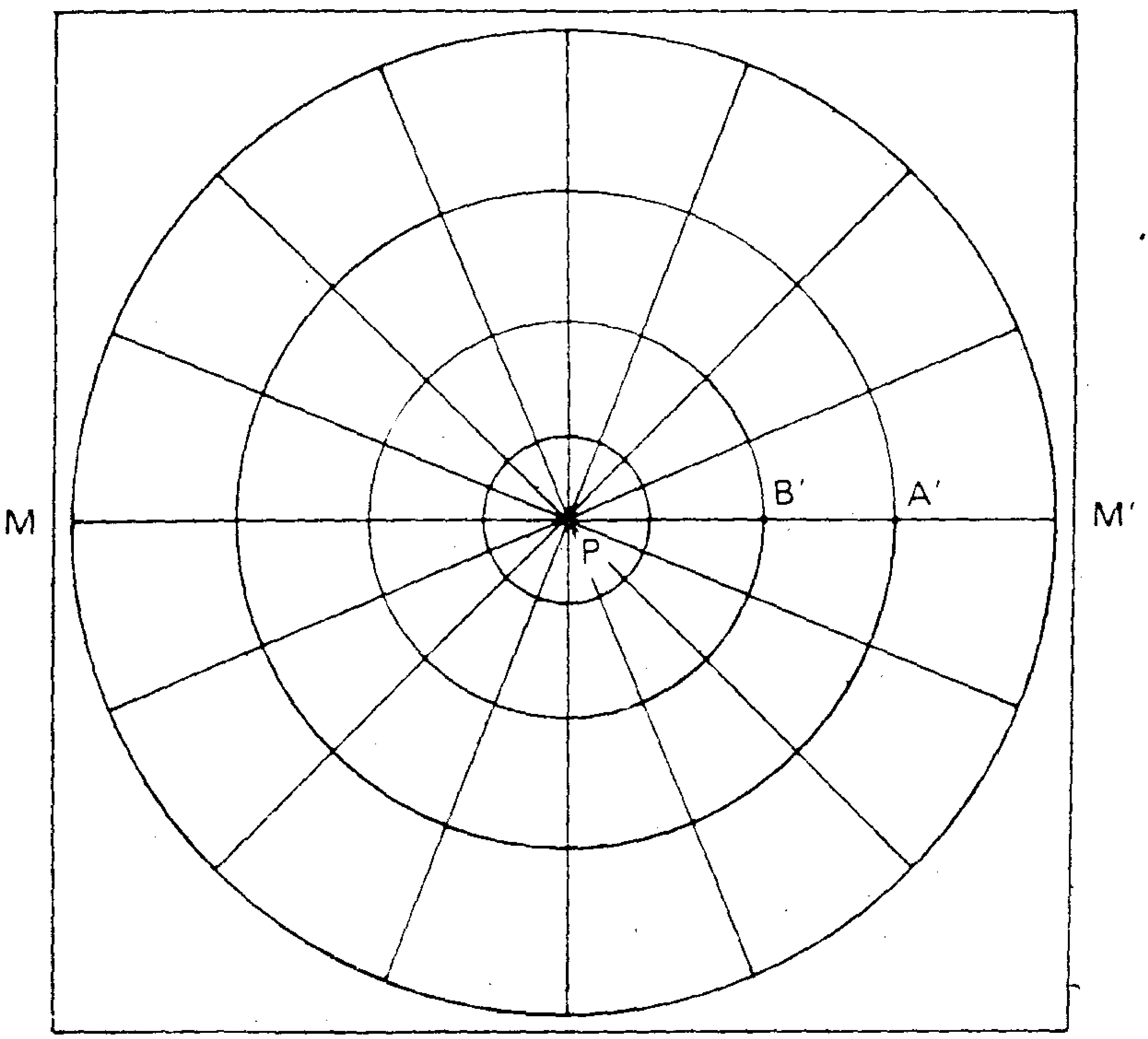


Fig. 8

would appear as straight lines crossing each other at the Pole, which is the tangent point. The parallels of latitude will appear as concentric circles with the pole as the centre, but with increasing radii as you go further away from the pole. (see fig 8)

If we consider any one meridian, we can show mathematically, the distance at which various latitudes will be projected. This is shown in the following illustration. (see fig 9)

Let us consider the meridian MM'. A & B are two places on the meridian.

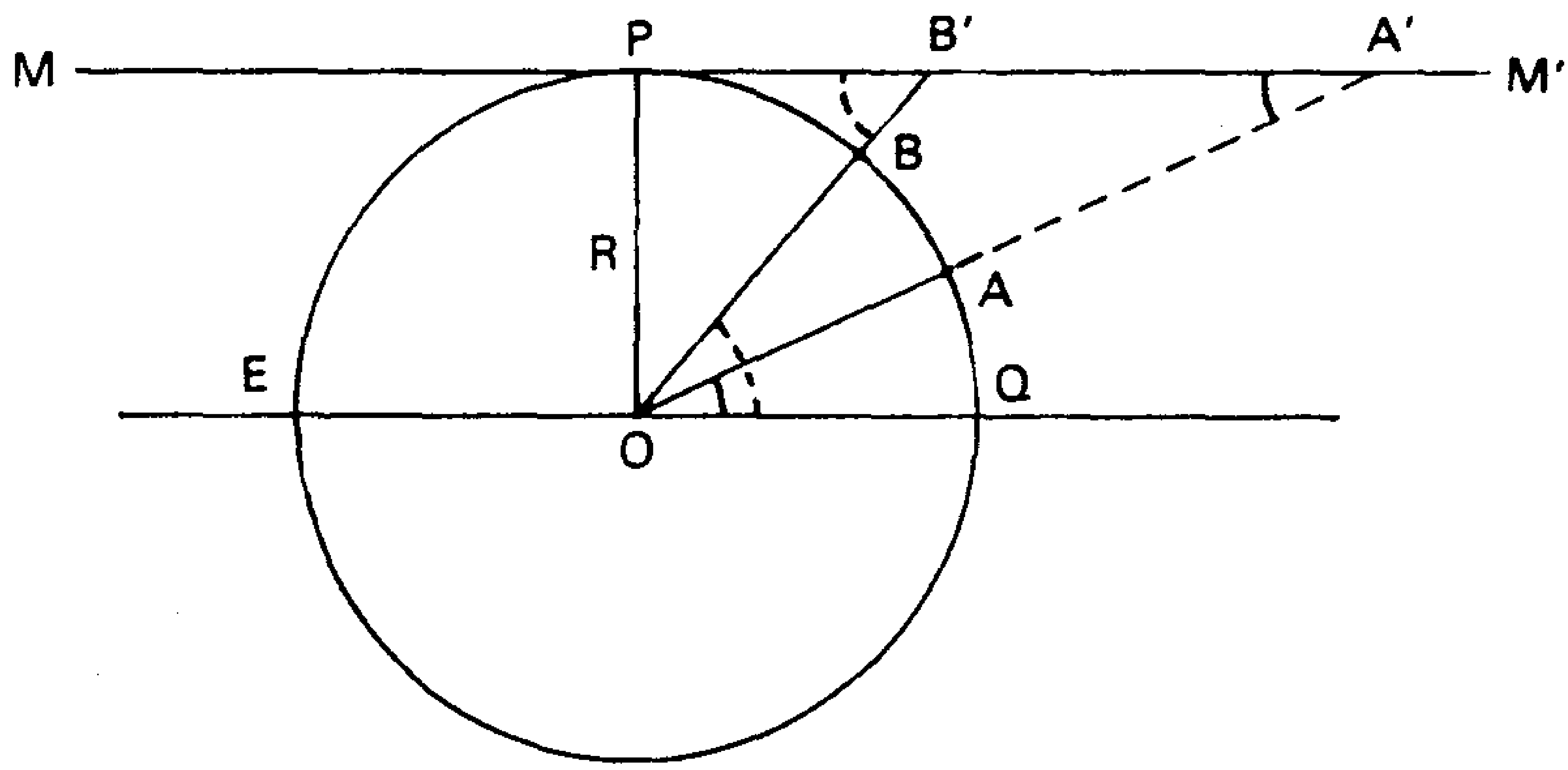


Fig. 9

In figure 9.

EQ - Plane of Equator.

MPM' - Projection of meridian EPQ

P - Pole (Tangent point).

O - Centre of the earth.

A & B - Two places on that meridian.

A' & B' - Points of projection of A & B respectively.

$A\hat{O}Q = P\hat{A}'O$ = Lat of A.

$B\hat{O}Q = P\hat{B}'O$ = Lat of B.

PO = Polar Radius of earth (R)

It can be seen from the figure that

PA' = R Cot. Lat of A

PB' = R Cot. Lat of B

Hence the Radius of the lat circles on the chart will vary as the Cotangent of latitude. When we come to the equator Cot of zero being infinity, the equitorial regions cannot be shown on this chart, as the distortion becomes excessive. Thus the maximum area that can be projected at one time is just under a hemisphere. The calculated radii of the various latitude circles on the chart, are reduced to a suitable scale before plotting them on the chart, In practice however, specified small areas on such a projection can be drawn to a larger scale, even though the tangent point may be outside the chart.

Advantages of a Gnomonic chart

(1) Lats & Longs can be readily lifted off the chart.

(2) Great circle courses can be drawn as straight lines joining the two positions.

(3) Courses can be read off at respective meridians using a protractor.

Disadvantages :

(1) No compass rows are shown on the chart, as the azimuths are correct only at the tangent point.

(2) Courses, bearings and position lines can not be transferred from one part of the chart to another.

(3) Distances cannot be readily measured.

(4) Rhumb lines cannot be readily shown.

Uses of Gnomonic chart :

Because of several disadvantages, a Gnomonic chart cannot be used by itself directly for the purpose of Navigation. A Gnomonic chart is always used in conjunction with a mercator's chart of the same area. Having obtained the G.C. Track from a gnomonic chart, by joining the departure and arrival positions by a straight line, which is the G.C. Track, Lats & longs of several convenient points along the track are lifted off this chart. These points are re-marked on a mercator's chart of that area. Short legs of rhumb lines are drawn between these points on the mercator's chart and the ship is sailed along these short rhumb line courses.

In the ultimate analysis, it will be seen that a ship is sailing along an approximate G.C. and not on an exact G.C. To steer exactly along a G.C. will make it necessary to continously keep altering the course of ship which is very impractical.

Exercise II

1. Explain the Principle Mercator's projection.
2. Given the longitude scale of a Mercator's chart as 10 cm to one degree of long, calculate the distance apart between the latitude of 22° & 24° on that chart.
3. Define Meridional parts. Explain how this can be used in constructing a Mercator's chart.
4. What is Natural scale of a chart ? Given the Long scale of a mercator's chart as 10 cm to one degree of longitude find the Natural scale in the lat of 54° on that chart.
5. Explain the principle of Gnomonic projection.
6. Given the Polar radius of the earth as 6356.5 km, find the radius of a circle of latitude 60° N on a Polar Gnomonic chart drawn on a scale of 10 cm to 1000 Km.
7. Why is it that a Gnomonic chart cannot directly be used for Navigation ? If so explain how that chart is used in practice, for Navigational purposes.

Answers

Q.2 21.6 cm

Q.4 $\frac{1}{662883}$

Q.6 36.7 cm.

CHAPTER III

THE SOLAR SYSTEM

The Solar System refers to the Sun and various planets and their satellites The centre of gravity of the entire system is near the centre of Sun. The Planets revolve round the Sun in well defined orbits. Similarly various satellites of the planets also revolve round their respective parent planets. In addition there are also various comets, which also go round the Sun in very elongated orbits. The whole system collectively is known as the Solar system.

The Planets in the order of distances from the sun are Mercury Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune & Pluto Of these only four are bright enough to be easily visible to the naked eye, and are used for navigational purposes. These are Venus Mars, Jupiter and Saturn.

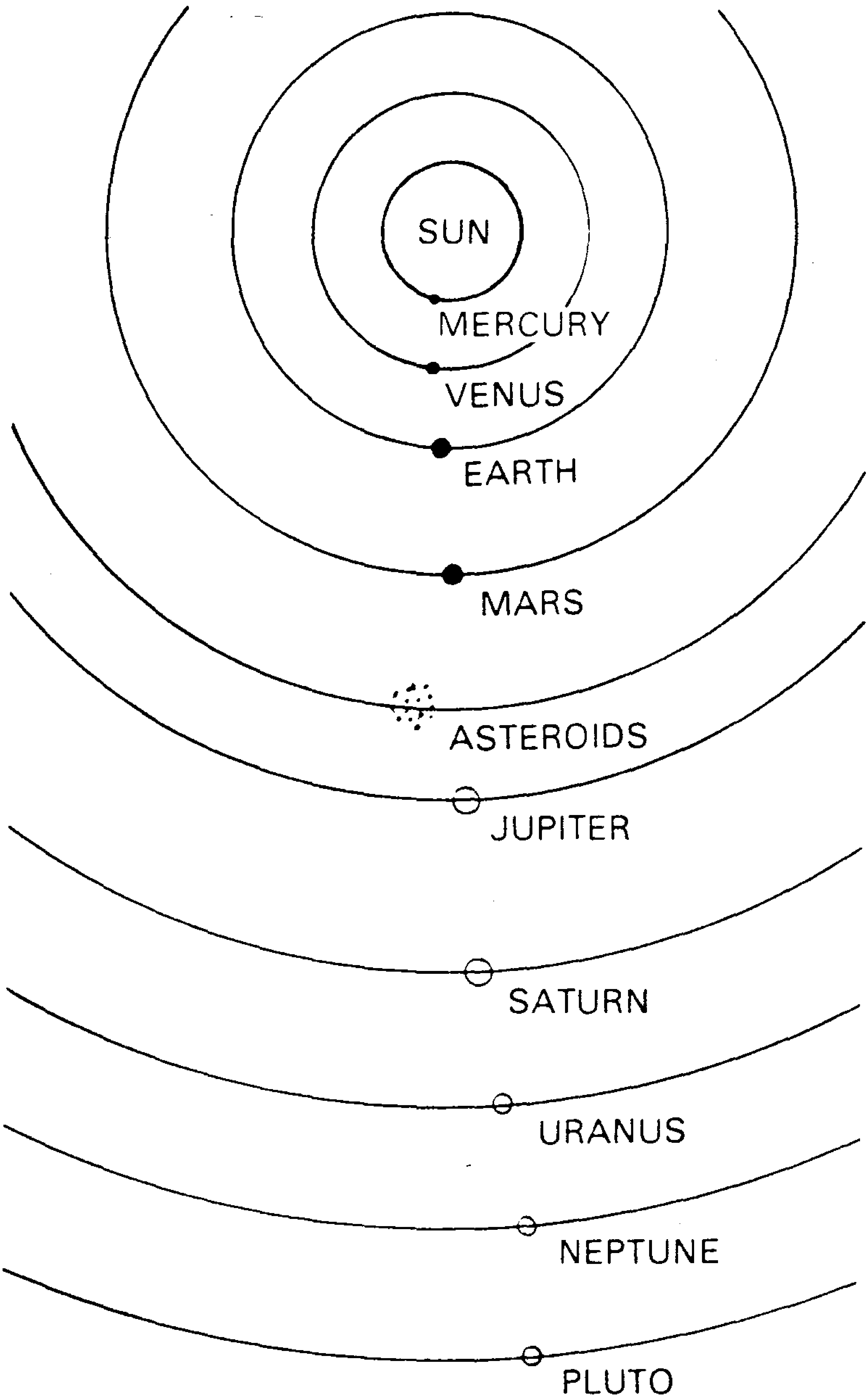
Between the orbits of Mars and Jupiter, there are multitudes of small planets collectively called Asteroids revolving round the Sun similar to other planets. These are believed to be broken pieces of some planet that had existed in this space earlier.

Planets do not emit any light of their own. But their presence in the orbit is seen, because of the sun's rays falling on them and they reflecting the rays to the earth. Hence they seem to be steady lights in the sky.

Stars however, emit their own light, and twinkle in the sky. It is quite possible that many of the stars we observe may have their own solar system Because of vast distances that seperate stars from us, little is known about the stars

Earth's closest neighbour in the sky is the Moon situated at an average distance of 240,000 miles from the earth. The mean distance of the sun from the earth is about 93 million miles. All planets with the exception of Venus & Mercury have their own moons or Satellites.

The following Table gives the sizes of planets and distances from the sun and other relevant parameters in the Solar system.



PLANETS IN SOLAR SYSTEM

Fig. 10

SOLAR SYSTEM

Name	Mean Diameter in Miles	Mean Dist. from Sun in millions of Miles	Time taken for 1 Rev.	No. of Moons
Sun	866,400	–	–	–
Mercury	3,000	36	88 days	0
Venus	7,000	67	225 days	0
Earth	7,927	93	365¼ days	1
Mars	4,200	142	687 days	2
Jupiter	88,250	483	11.86 Yrs	11
Saturn	74,240	886	29.4 Yrs.	10
Uranus	32,000	1782	84.0 Yrs.	4
Neptune	32,900	2792	164.8 Yrs	1
Pluto	–	3701	248.0 Yrs	1

The moon takes 27⅓ days to go through 360° of its orbit round the earth. Its diameter is 2164 miles.

All the bodies that go round the sun have two types of motion – They go round their respective axis. This motion is called 'Rotation'. They go round the sun in well defined orbits. This orbital motion is called "Revolution". The sun also rotates on its axis once in 25 days. All the bodies are kept in their relative positions and in their respective orbits by definite mathematical laws. All bodies exhibit a gravitational attraction of each other. According to Newton's Law this attraction is directly proportional to their masses and inversely proportional to the square of the distance that seperates them.

Kepler's Laws: The orbital motion of bodies is governed by Kepler's laws of planetery motion. These laws are –

1) The orbits of all bodies around the central body are elipses, with the central body situated in one of the focii.

2) For a body in orbital motion, the radius vectors sweep out equal areas in equal time.

3) $\frac{T^2}{d^3}$ for all bodies is a constant.

Where 'T' is the sideral period of the body (ie. time taken by the body to go round the central body, exactly through 360°. 'd' is the mean distance from the central body.

Ellipse : An ellipse can be drawn, by fixing two pins A & B and a piece of string looped over the pins, with a pencil point held in the loop, and tracing a figure on the paper by keeping the string tight. The resulting figure is an ellipse.

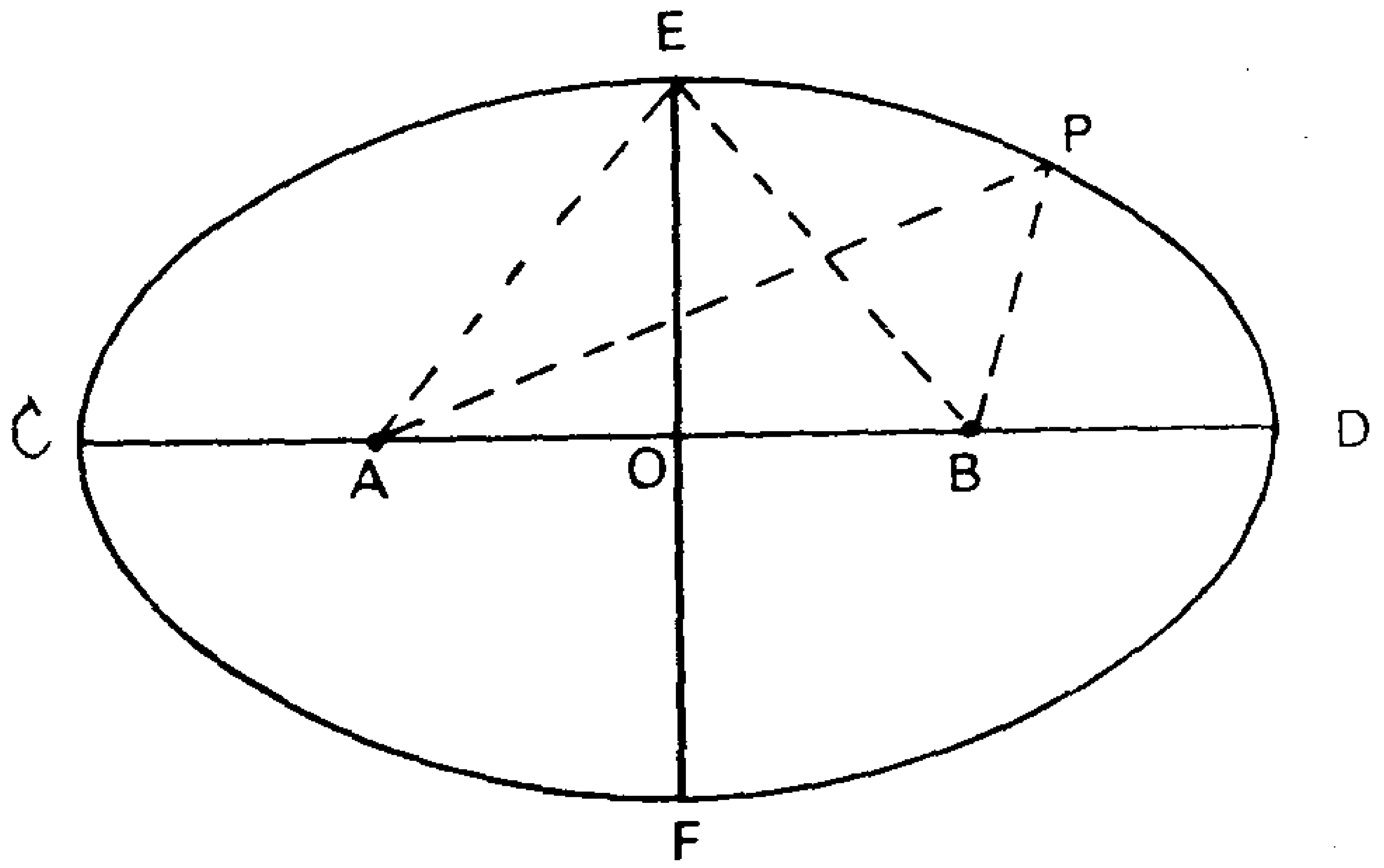


Fig. 11

Each of the two points A & B is called a focus of the ellipse. An ellipse has two focii, unlike a circle which has only one centre. For any point P on the circumference of an ellipse, the sum of the distances to the focii is constant.

CD is called the major axis, & EF, the minor axis. OC or OD is the semi-major axis, & OE or OF is the semi-minor axis.

The Ratio $\frac{OB}{OD}$ or $\frac{OA}{OC}$ is called the eccentricity of the ellipse.

This ratio determines the shape of the ellipse. Smaller this ratio, more elongated the ellipse will be. As this ratio approches, unity, the shape of the ellipse approaches a circle.

The meaning of the 2nd law of Kepler's is illustrated in fig. 12 If 'S' represents the central body situated at one of the focii, and a revolving body moves from A to B in one day thus sweeping out area SAB, this area swept out will remain constant through out its revolution provided the interval is constant. It means the body will move from C to D in one day when it is nearest to 'S' and sweep out an area CSD.

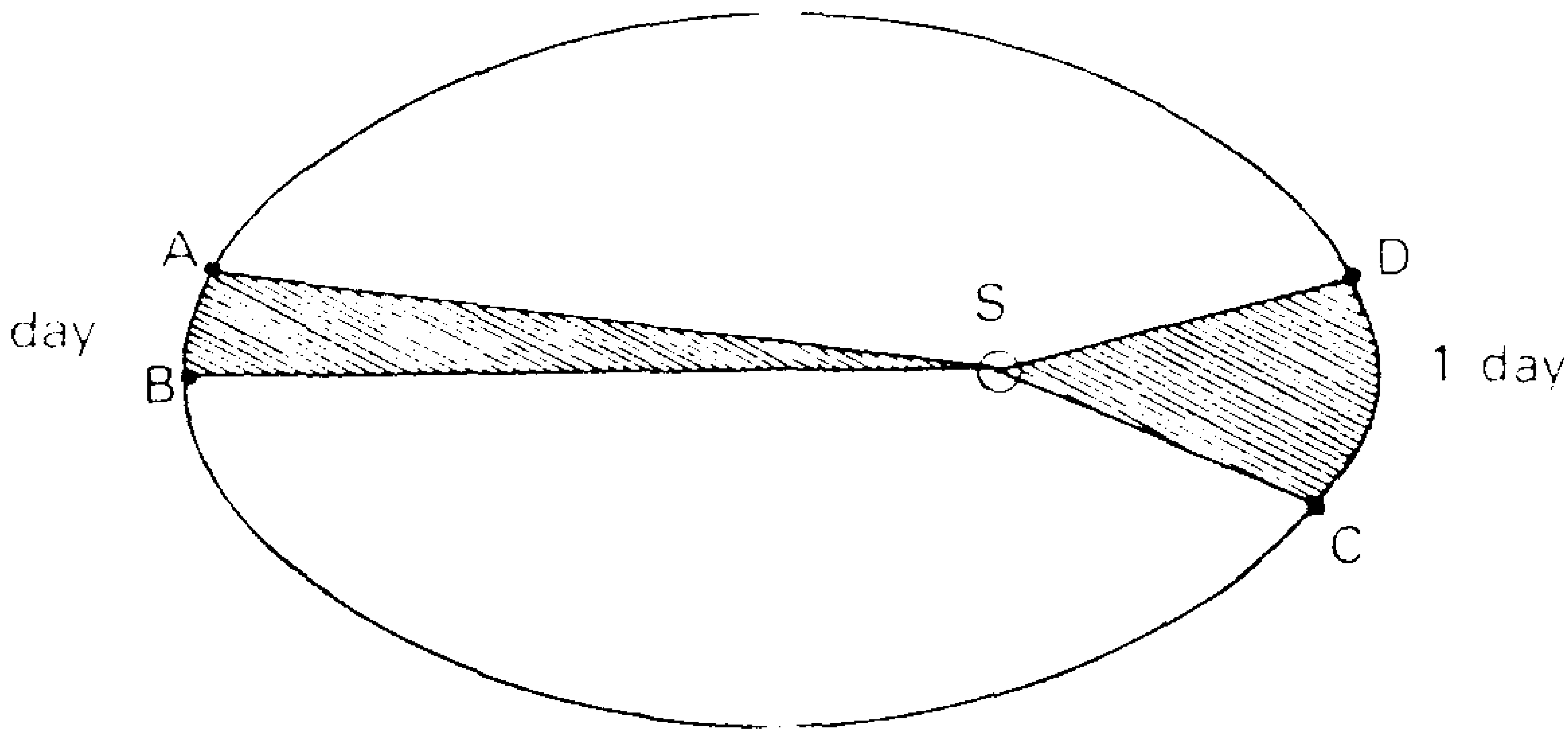


Fig. 12

By this law area SAB = area CSD.

Thus arc CD is greater than arc AB. Hence it follows that when the revolving body is closest to the central body, it moves fastest on its orbit and when it is farthest away from the central body, it moves slowest. If we apply this to the earth/sun system, (see fig 13) when the earth is at E it is farthest from the Sun at about 95 million miles and is said to be in **Aphelion**, which occurs around 1st of July each year. The earth is now moving at its slowest orbital speed. When the earth has moved to E_1 it is closest to the Sun, at about 92 millions miles and is then said to be in **Perihelion**. This occurs about 1st of January each year. The orbital motion of earth will be fastest at perihelion. The line E to E_1, if indefinitly produced on each side is called the **Line of Apsides**.

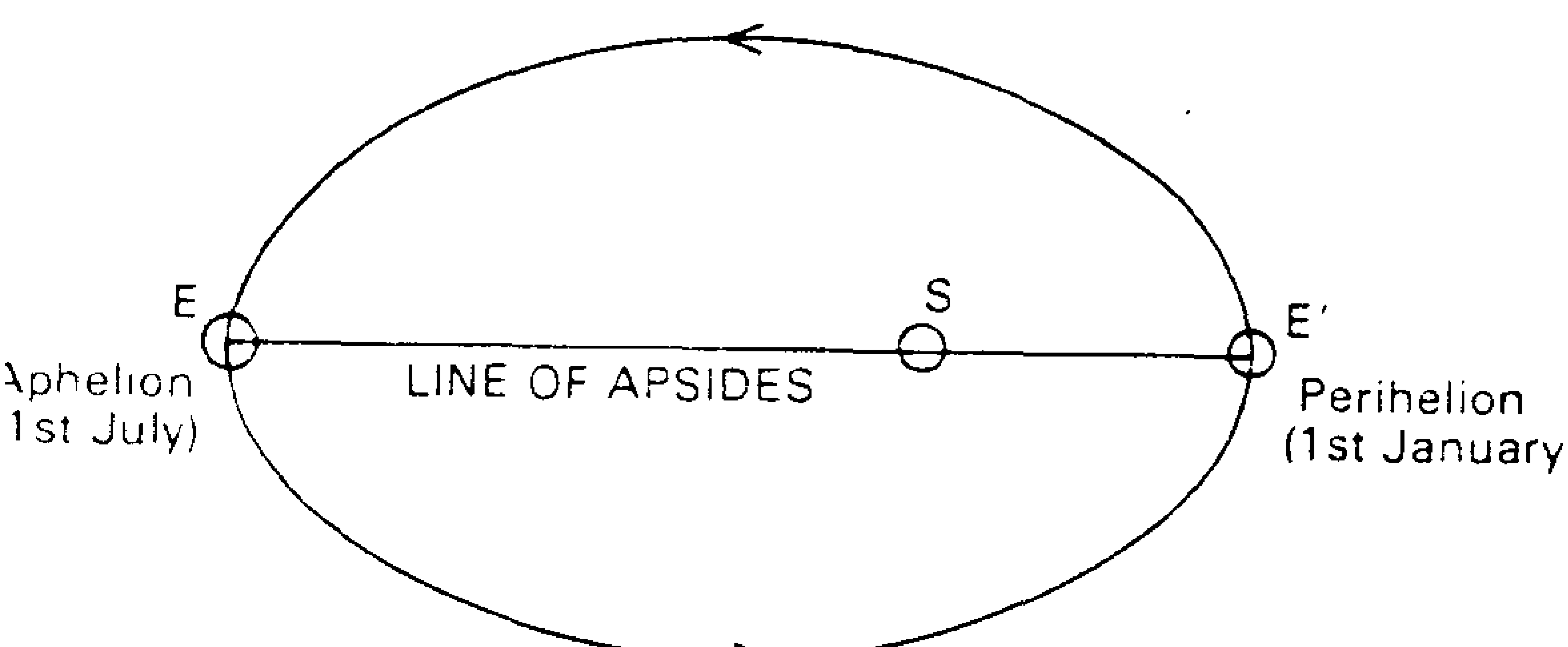


Fig. 13

The moon's orbit around the earth is also elliptical when the moon is closest to the earth it is about 221000 , miles away and it is said to be in **Perigee** and when the moon is farthest from the earth it is about 253000 miles and is said to be in **Apogee**.

Exercise III

(1) Write down the names of the planets in the Solar system in the ascending order of distances from the sun.

(2) State Kepler's laws of planetary motion.

(3) How will you construct an ellipse & what are its properties ?

(4) Define the following terms :-
(a) Eccentricity of an ellipse (b) Aphelion (c) Perihelion
(d) Line of Apsides (e) Apogee (f) Perigee.

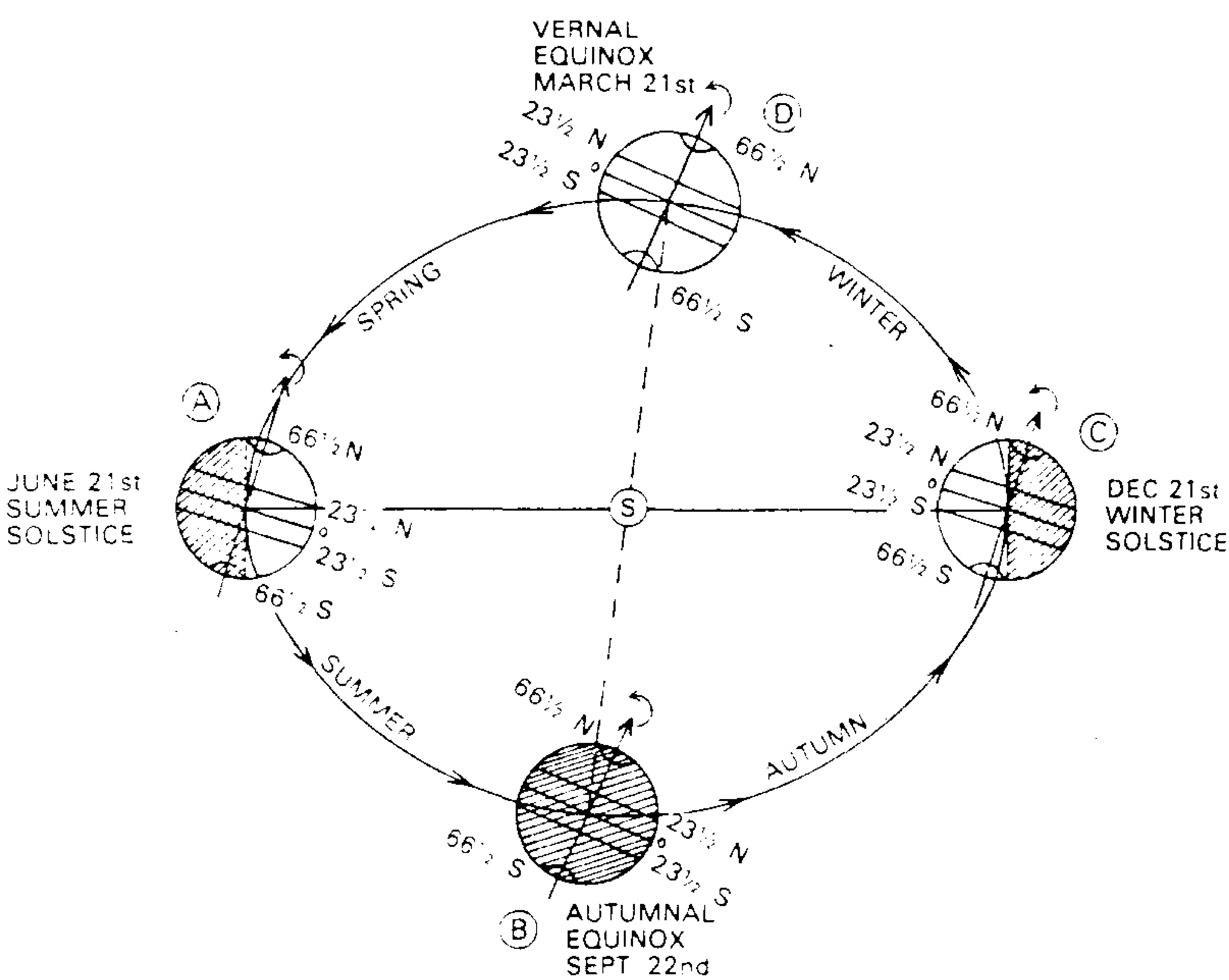
(5) Why does the earth have slowest orbital motion at aphelion and fastest motion in perihelion and when do these occur in the year?

CHAPTER IV

DAY & NIGHT & SEASONS

The movements of the earth and the consequent effects are of particular interest to us on the earth.

Like all other bodies, the earth rotates on its axis once in 24 hours and simultaneously it also has a motion on its orbit, which it completes once in 365¼ days ie. in one year. The earth's axis is not perpendicular to the plane of its orbit, but is tilted to an angle of 23½° to the plane at right angles to its orbit and keeps spinning continously in an anticlockwise direction as viewed directly on the N. Pole from outer space. In other words, the direction of rotation of the earth is from West to East. This is a **True Motion.** Due to this rotation, all heavenly bodies appear to rise in the East, go across the sky, and set in the West. The passage of a heavenly body from rising to setting across the sky



UNEQUAL LENGTH OF DAY & NIGHT & SEASONS

Fig. 14

is called the **diurnal motion** of the body, which is an **Apparent Motion.** During the diurnal motion of body, when it reaches the observer's meridian, it attains its highest altitude above the horizon for the day and this phenomenon is sometimes referred to as **culmination** but commonly called the **meridian passage.**

Figure 14 represents four specific positions of the earth on its orbit around the Sun. It may be observed that the earth's axis is kept pointing in the same direction in space irrespective of the earth's position in the orbit. The spinning earth is like a spinning gyroscope and exhibits the same fundamental properties. Because of the inherent property of rotational inertia the axis keeps pointing in the same direction in space.

Day & Night

The Sun situated in one of the focii of the elliptical orbit shines directly on the earth, and as the earth rotates on its axis various places on the earth's surface are brought in front of the sun, and some places are rotated away from the sun's rays. At any instant of time, the sun's rays light up exactly half the earth's surface. That half turned towards the Sun experiences the day, and that half turned away from the Sun experiences the night. The total of the hours of light and darkness together make up 24 hours of the day in which time the earth completes one rotation on its axis.

The duration of day light and darkness are unequal in most parts of the earth.

Consider, the position of the earth at 'A' shown in the fig. 14. Though the sun is lighting up exactly half the surface of the earth, the direct line joining the sun to the earth meets the earth at 23½° N Latitude and continues to 23½° S Latitude. This direct line, in the same plane as the orbit, is the plane of the **Ecliptic.** It may be observed that the spin axis of the earth is titled at any angle of 23½° to a plane at right angle to the orbit. The equator is also making the same angle with the plane of ecliptic and this inclination of the ecliptic to the equator is called the **Obliquity of the ecliptic.**

Unequal length of day & night

Refering to fig. 14 the position of the earth at A occurs on June 21st when the Sun is shining directly over 23½° N latitude. This latitude is called the **Tropic of Cancer.** If, on this day, an observer were to travel from the equator to the North Pole it will be observed that as he increases in latitude, greater and greater arcs of N. Latitudes are turned towards the sun, & shorter & shorter arcs of corresponding latitudes are turned away from the sun, thus indicating the length of day light hours are longer than night. The length of the day keeps on increasing till lat. 66½°N is reached. Any latitude above this is tilted towards the Sun, thus experiencing daylight throughout 24 hours and the Sun does not rise or set.

Reverse phenomenon takes place in South latitudes on this day. If the observer travels towards South pole from equator, he will observe that greater and greater arcs of latitudes are turned away from the Sun as he increases in latitude till he reaches 66½° S, and beyond that the entire latitudes are turned away from the Sun.

This illustrates that, the length of the night is longer than the day & beyond 66½° S. it is night for all 24 hours and the sun does not rise at all.

Thus on June 21st the sun has reached its maximum North Declination and all Northern latitudes will experience the longest day and shortest night whereas, the Southern latitudes will experience, their longest night and shortest day. This phenomenon is termed **"Summer Solstice"**.

Since half the equator is turned towards the sun always, there is equal length of day and night on all day at the equator.

The earth moves in the orbit day after day. After 3 months it would reach a position "B" (fig. 14). Now the Sun will be shining directly at the equator.

To an observer on the earth it would appear as if the Sun has gradually changed its position along the line 'EC' to be at 'O' on the equator on this day. (See fig. 15) This occurs of 22nd September, and all places on the earth's surface will be having equal length of day and night. This day is called the **Autumnal Equinox**. The term "Equinox" means equal length of day & night.

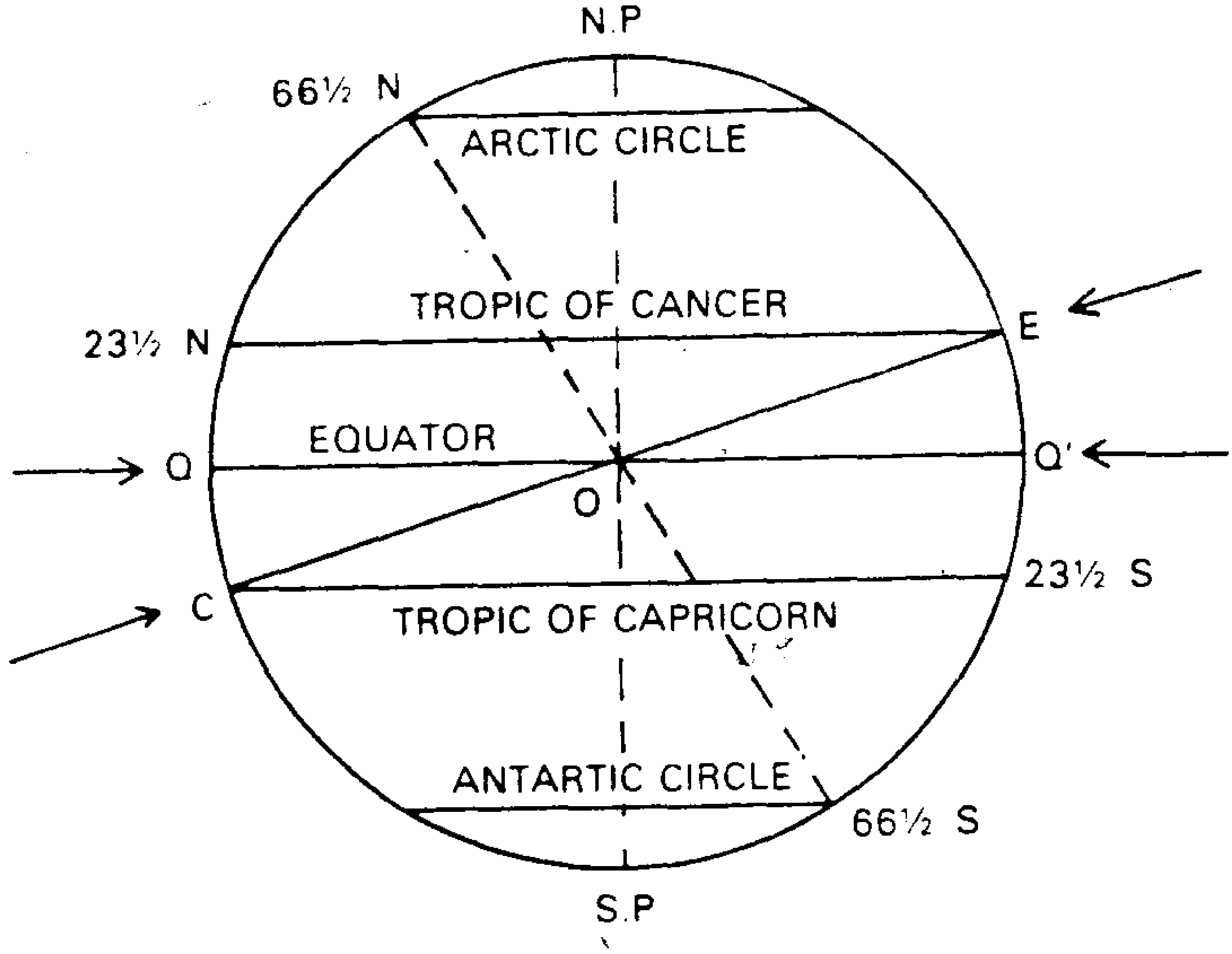


Fig. 15

Three months later the earth would have reached a position 'C' in fig. 14. To any observer on the earth, it would have appeared as if the Sun gradually changed its position daily from O to C in fig. 15 and would now be shining directly over the latitude C ie. 23½° S which is called the **Tropic of Capricorn.** On this day if one was to travel from the Equator to the South Pole, as he increases in latitude, more & more arcs of latitude will be turned towards the Sun & shorter arcs will be turned away from the Sun. This indicates that all places in south latitude are experiencing longer days and shorter nights. The reverse phenomenon will occur in Northern hemisphere Longer arcs of latitude are turned away from the Sun, & shortest arcs turned towards the sun, indicating, longer nights & shorter days. At the equator, however, there will be equal length of day and night, as exactly half the arc of the equator is turned towards the sun. This position of the earth is called the **"Winter Solstice"** and occurs on 21st December, when longest day will be experienced in Southern hemisphere.

Beyond 66½° S, the Sun will not set at all & will remain above the horizon all day. In the Northern hemisphere, however, beyond 66½° N, the Sun will not rise at all, and hence no daylight and all 24 hours cf the day will be night.

After a further lapse of three months, the earth would reach position 'D' in Fig. 14 when the sun will again be shining directly on the equator, thus producing equal length of day & night in all latitudes. This occurs on March 21st and the day is called the **"Vernal Equinox"**.

Three months later the earth returns to position 'A' in Fig. 14 thus completing the revolution round the Sun, which takes 365¼ days ie. one year The cycle repeats over again year after year.

Seasons : Due to the revolution of the earth round the sun, the sun appears to change in declination and trace out a path on the earth surface along the plane of the ecliptic. When the sun is shining directly over the Northern latitudes ie. when it has Northerly declination, the Northern hemisphere experiences **"Summer"** and Southern hemisphere experiences **"Winter"**. Six months later, when the position is reversed, ie. when the Sun is shining directly over Southern latitudes ie. when it has a Southerly declination, the Southern hemisphere experiences "Summer" & Northern hemisphere, the "Winter". The transition from summer to winter is called **"Autumn"** and winter to summer is called **"Spring"**. The path of the earth round the sun is thus divided into four quarters as shown in fig. 14 to indicate the respective seasons. It may be observed that the seasons indicated therein are those experienced in Northern hemisphere, and in the Southern hemisphere it will be just the opposite.

Climatic Zones : (see fig. 16) Since the declination of the Sun changes from 23½°N to 23½°S & vice versa, the Sun will shine directly overhead at any place within these latitude twice in a year, once when it is moving South and once while it is moving North. What is more, the hotest part of the earth will also be within these latitudes. This area is therefore called the **"Torrid Zone."** Between 23½° N or S & 66½ N or S, though the sun will rise & set everyday, it can never be overhead at any time. Hence the climate will be more moderate and so this area is termed the

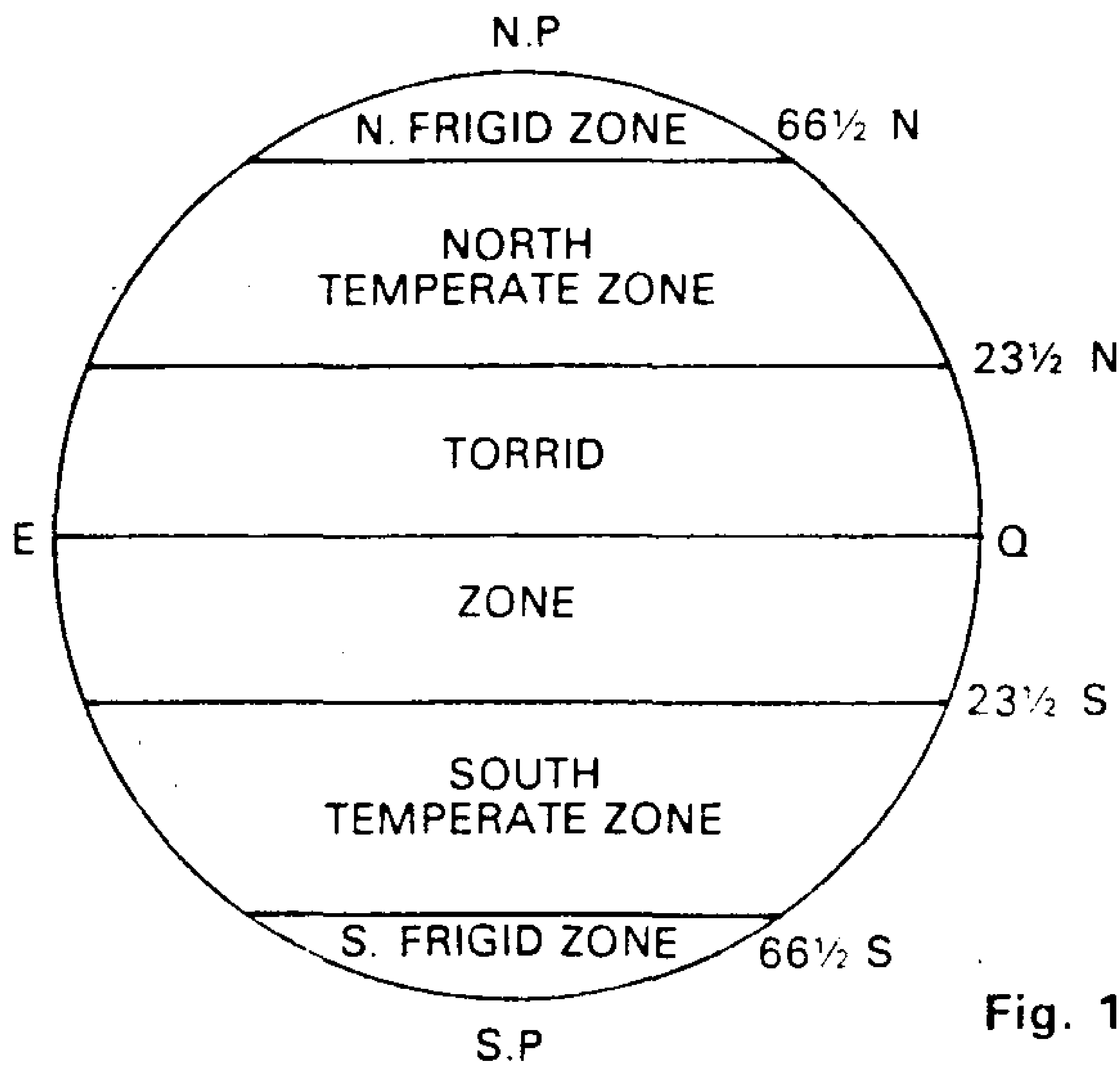


Fig. 16

"**Temperate zone**". Beyond 66½° N or S, the Sun is below the horizon for six months in the year. For the remaining six months, though the sun remains above horizon all day, it is so low in the horizon, that even at noon its ray are so slanting that it does not convey much heat to the surface. Thus this area remains rather cold throughout the year and hence named as the "**Frigid zone**".

Exercise IV

(1) What are "True & Apparent motions" of heavenly bodies ? Give one example of each.

(2) How are day and night caused ? How and why does the duration of day light hours vary throughout the earth ?

(3) When and where will the length of day & night be of equal duration on the earth's surface?

(4) Define the following terms :- Ecliptic, obliquity of the ecliptic, Equinoxes, Solstices, Torrid zone.

(5) Explain with the help of a diagram, how seasons are caused on the earth's surface.

* (6) In a certain Latitude, the ratio of the longest day to shortest day is 3 : 1 - find the latitude. (Ans. 58° : 24.7° N or S)

* This question should be attempted only after understanding solution of quadrantal triangles discussed in chapter XIII.

CHAPTER V

CORRECTION OF ALTITUDES

In all navigational problems, it is necessary to correct the altitude of a heavenly body taken with a sextant, to True Altitude of the body before it can be used to solve the problems. This is because of the fact, in solving the astronomical triangle PZX, all sides have to be arcs of Great circles. The sides PZ & PX are already arcs of Great circles,being arc of meridians, whereas the side ZX will become an arc of a great circles, only after correcting the observered altitude to True Altitude.

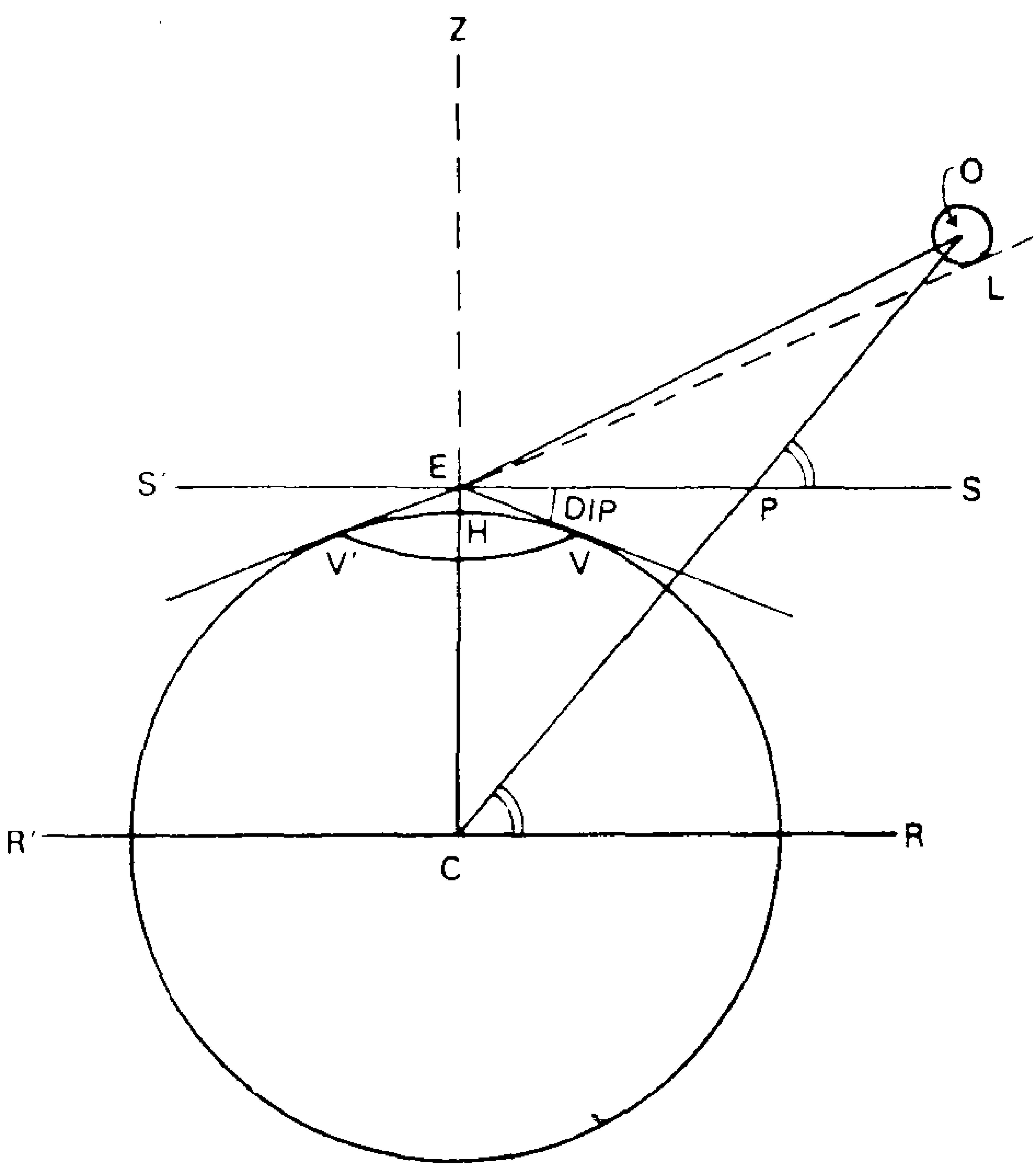


Fig. 17

The figure is drawn on the plane of the vertical circle passing through the body 'O'.

Definition : Visible horizon is the circular line binding the observer's view at Sea. (VV' in fig. 17) HE represents the height of Eye of the observer above the earth's surface. Higher the observer, further away will the visible horizon be.

Sensible Horizon is an imaginary horizontal plane passing through the observer's eye (SES' in fig. 17).

Rational Horizon is an imaginary horizontal plane, parallel to the sensible horizon and passing through the centre of the earth. (RCR' in fig. 17).

When an observer takes an altitude of a body, he is in fact measuring the angular height of the body above his visible horizon at sea. His eye itself is situated at some height above the surface of the earth.

Planets and stars are merely points of light and do not present any visible disc. The sun & moon do present a visible disc and it is therefore difficult to judge, where exactly their centre is and so we take the altitude either of the lower limb (LL) or the upper limb (UL).

Lower limb is that arc of the visible disc, closest to the horizon used.

Upper limb is that arc of the visible disc, farthest from the horizon used.

The centre of the body will be exactly half its diameter, called the "Semi-diameter" away from the Limb observed. If LL is observed, the SD is additive to the altitude and if UL is observed the SD is subtracted from the altitude to get the altitude of the centre. This correction is not applicable to the altitudes of planets & stars as they are only points of light.

Of all the altitudes we observe the altitude of the moon, has the maximum number of corrections. Therefore the altitude correction of the Moon is discussed in detail.

Correction of Altitude of the Moon (Refer to fig. 17)

Sextant Altitude	=	$L\hat{E}V$
Index error	=	+ off the arc (–on the arc)
Observed altitude	=	$L\hat{E}V$
Dip for HE	=	$-S\hat{E}V$ (always – ve)

Apparent altitude of LL	=	$L\hat{E}S$
Refraction	=	-ve (always - ve)
Apparent Alt of LL	=	$L\hat{E}S$
Corrected S.D.	=	Arc OL (LL + UL -)
Apparent Alt of centre	=	$O\hat{E}S$
Parallax-in-altitude	=	$E\hat{O}C$ (Always + ve)
True Altitude	=	$O\hat{C}R$

It will be seen from figure that $O\hat{E}S + E\hat{O}C = O\hat{C}R$

Sextant altitude is the altitude as observed on the Sextant ($L\hat{E}V$)

Index error if any, is a small instrumental error of the sextant. "Off the arc" errors arc + Ve and "On the arc" errors arc - Ve *(Being very small, not specifically shown in figure)*

Observed altitude is the altitude of the LL or UL above the visible horizon ($L\hat{E}V$ in fig).

Dip or depression is the angle of depression of the visible horizon below the sensible horizon. This is always a negative correction and is to be obtained from the nautical tables for the respective height of eye. ($S\hat{E}V$ in fig).

Apparent Altitude : is the altitude of the point observed above the sensible horizon. ($L\hat{E}S$ or $O\hat{E}S$ in fig).

Refraction : The earth's atmosphere through which the ray of light from a body passes, has density. Because of this density, a ray of light when passing through the atmosphere is bent towards the normal before reaching the observer's eye. as shown in fig. 18. A body situated at a point X will appear to be positioned at X' to an observer. The effect of refraction therefore is to make the body appear higher than where it actually is. Hence this correction is always subtractive, to bring the body to its correct position. The value of refraction is tabulated in any nautical tables for standard atmospheric conditions which is taken as a pressure equivalent to 29.6" of mercury and 50° (F) temp. For any variations from the standard conditions a supplementary correction is necessary which is also tabulated in the adjacent tables to the main refraction tables. This supplementary correction is to be applied to the main correction as per sign given before applying it to the altitude. It will also be observed from these tables that refraction is maximum (about 32′ of arc) at zero degrees altitutide and nil at 90° altitude. Hence refraction from the tables has to be taken for the appropriate apparent altitude.

Being a very small correction it is not specifically indicated in fig. 17.

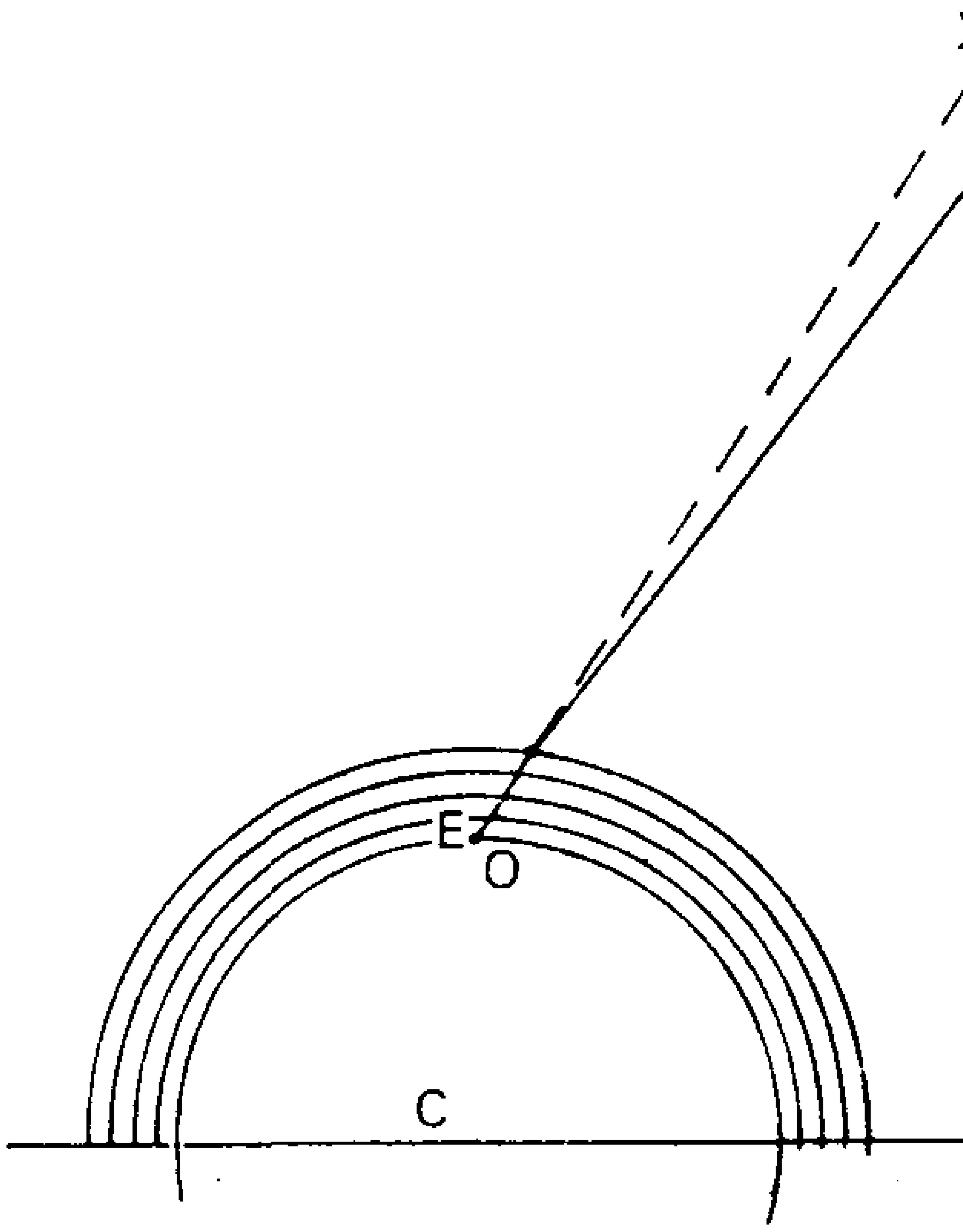


Fig. 18.

Semi-Diameter is only applicable to the altitudes of Moon and Sun.

In the case of the sun, the S.D. is tabulated for the middle day of the page in the nautical almanac alongwith the sun's elements. This is the S.D. for zero hours GMT on that day. If there is a change of S.D. from day to day, then the S.D. has to be increased or decreased proportionately for the actual GMT of observation before applying the correction to the altitude.

In the case of the moon, there are three values of S.D. tabulated in the nautical almanac under the moons' elements. The S.D., in the order in which they appear, is the S.D. for zero hours GMT on the first, second & third day respectively on that page in the nautical almanac. This is also the S.D. when the moon is on the observer's horizon. Since the observer is situated on the surface of the earth, the moon will be farthest away for him when on the horizon and will be at minimum distance when on his zenith. There is thus an apparent increase in the S.D. as it rises in altitude. The S.D. is least when on the horizon and maximum when altitude is 90°. This supplementary correction that is applied to the tabular S.D. is called **"Augmentation to Moon's S.D."** and is tabulated as a supplementary correction table in any nautical tables. This apparent increase is illustrated in fig. 19.

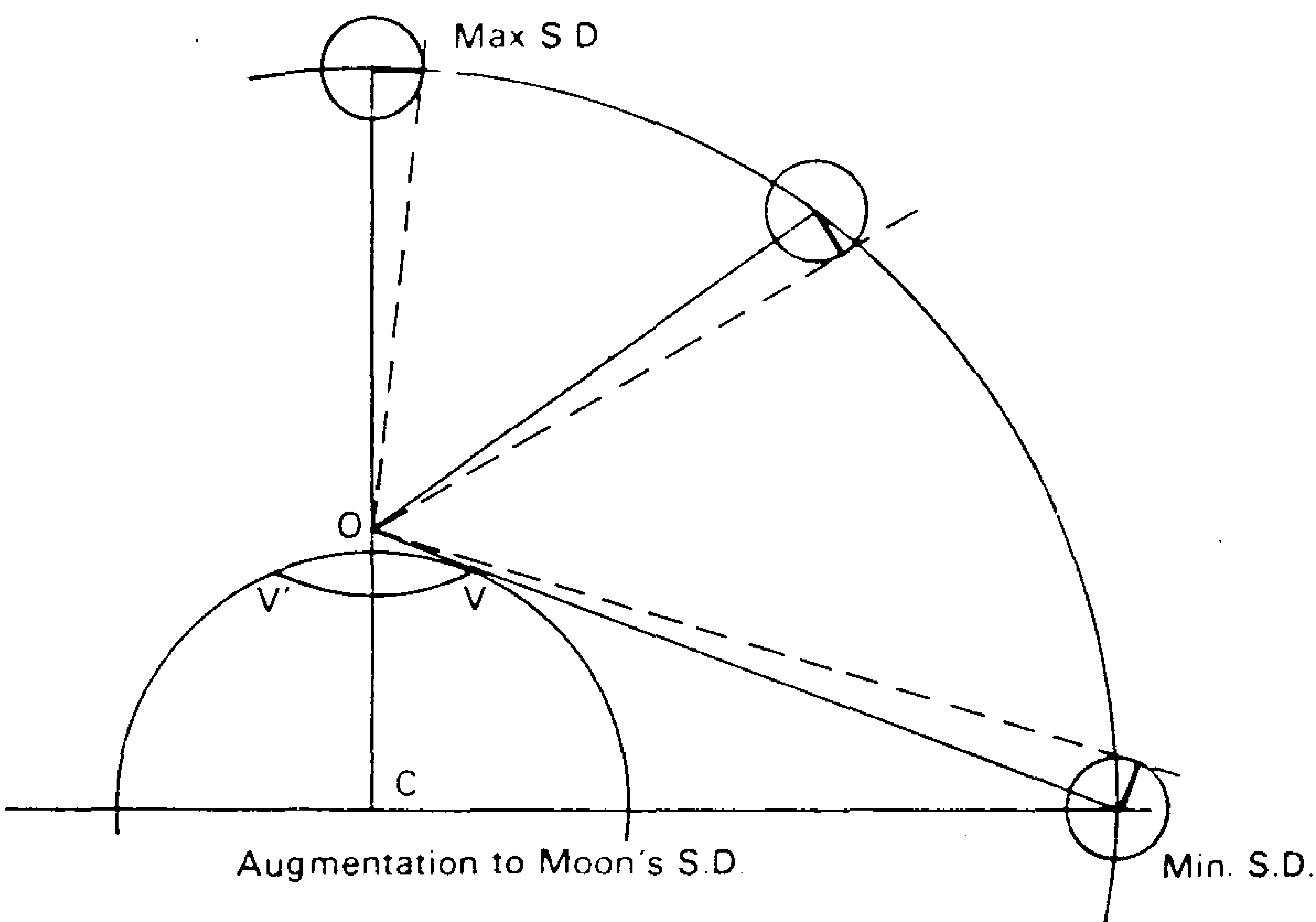


Fig. 19

Correction to apply to tabular S.D. from nautical almanac, before applying to altitude are :

S.D............................(*from Almanac*)

Correction for GMT of obsn...	.... + / .... −	*Proportionate change*
S.D. for GMT..................		− − − −
Augmentation.................		(+*ve*)
Corrected S.D................		(*Arc OL in fig. 17*)

Augmentation to the S.D. is only applicable to Moon's S.D. This is because, the moon being a body fairly close to the earth the apparent increase in the diameter, due to changes in altitude caused by the observer being situated on the surface of the earth, is a measureable quantity. Had the observer been situated at the centre of the earth, there would have been no augmentation correction for the moon.

In the case of the sun, however, even though the observer is on the surface of the earth, the Sun being so far away, the displacement of the observer to the surface of the earth, does not produce any measureable changes in the diameter of the Sun as it changes in altitude. Hence this correction is not applied in the case of the Sun.

This corrected S.D. of the moon, when applied to the apparent altitude (LL.+ ve, UL - ve), we obtain the apparent altitude of the centre of the Moon ($O\hat{E}S$ in fig. 17).

Parallax-in-altitude is defined as the angle subtended at the centre of the heavenly body, between the observer's eye and the centre of the earth, when the body has some altitude.

Parallax is dependent on two parameters : These are :
(1) distance the body is away from the observer.
(2) the altitude of the body.

(1) Distance : As a body gets further and further away from the earth the angle at the body between the observer on the surface of the earth and the centre of the earth gets smaller and smaller, Thus in the case of stars and the distant planets there is no parallax at all. The stars are several light years away from the earth. Hence the displacement of the observer to the surface of earth does not produce any angle at all at the body. The rays received on the earth will be parellel irrespective of whether the observer is on the surface of the earth or at the centre. But in the case of the moon, which is closest heavenly body to the earth, and the Sun and the two neighbouring planets to the earth viz Venus & Mars, there is a correction to apply for parallax.

Moon's Horizontal Parallax : As the name implies, Horizontal Parallax (HP) is the parallax of the moon when it is on the observer's Rational horizon. Along with the moon's elements,

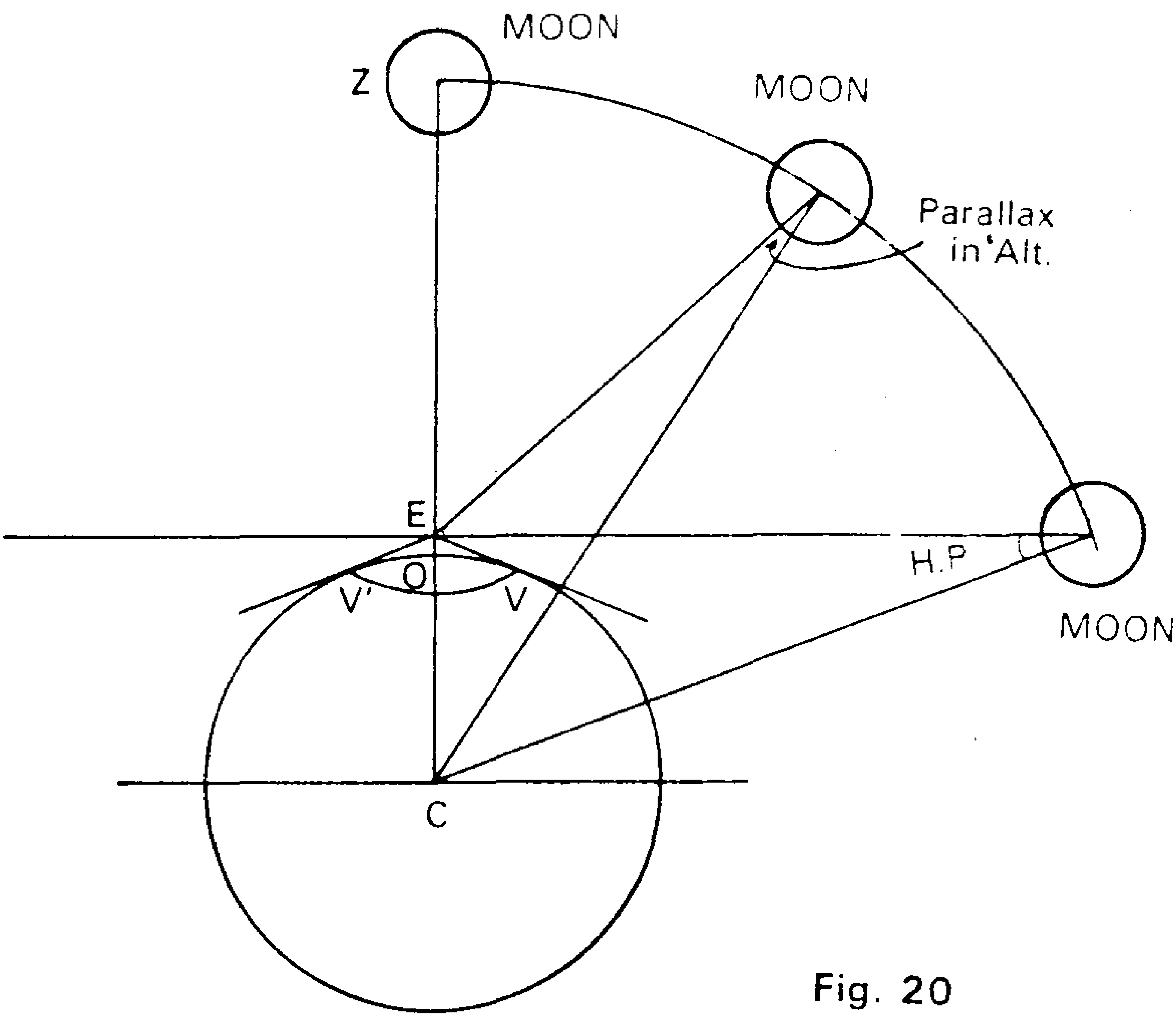


Fig. 20

ɪ P is tabulated for each hour of GMT in the nautical almanac. ˉhis tabular value is for an observer on the equator.

The earth being not a true sphere, but bulged at the equator ınd flattened at the poles, an observer at the equator will be ·losest to the moon when it is on his rational horizon. To an ıbserver at the poles, since he is closer to the centre of the earth, he moon will be further away for him when it is at his rational ıorizon. Since the H P tabulated is for an observer at the quator, the tabulated value of H P in the almanac is the ɿaximum possible value. As the observer increases in ɜtitude, he is getting closer to the centre of the earth and hence ɹrther away from the Moon and hence H P for such an observer ɿill be lesser than the tabular value. This supplementary correc- on we apply to the tabulated H P is called the **"Reduction to I P"** This is tabulated in any nautical tables with H P and atitude as arguments for entering the tables. From the above iscussion we see that H P taken from the almanac is subjected ɔ two corrections before we can put them to use. The method is hown below.

H P from Almanac for Whole No of hours of GMT	H P (Tabular)
Correction for Hly change (if any)	±(Proportionate change)
H P for GMT	– – – – –
Reduction to H P	– ve (from naut. tables)
Corrected Final H P	– – – –

(2) **Altitude** : This corrected HP will have to be converted to parallax-in-altitude before applying to the apparent altitude.

Parallax-in-altitude = H P × Cos. of Apparent Altitude.

It will be seen from figure, 20, that when the body is on the observer's Horizon, ie. when altitude is zero the

Pax-in-altitude = H P

ie. Pax in altitude = H P x 1 ....Eq 1

When the body reaches the Zenith ie. when the altitude is 90°

Pax-in-altitude = Zero.

ie Pax-in-altitude = H P x O ....Eq. II

comparing equations I & II we see that

Pax-in-alt = H P x a variable quantity between 1 & 0.

This varying quantity is the Cosine of App. Alt.

Cos Zero (Alt at Horizon) = 1

Cos 90° (Alt at Zenith) = O

∴ We can say parallax-in-Alt $\propto$ Cos app alt.

ie. parallax-in-alt = H P x Cos App. Alt.

We can now apply this Pax-in-Alt ($E\hat{O}C$ in fig. 17) to the apparent alt ($O\hat{E}S$ in fig. 17) to make it the true altitude ($O\hat{C}R$ in fig. 17).

Parallax-in-alt is always additive to apparent alt to make it the true altitude.

True altitude : is defined as the angle at the center of the earth or the arc of the vertical circle passing through the body between the Rational horizon and the line joining the body to the earth's centre. Being an angle at the centre of the earth it is now an arc of a great circle.

Zenith distance : is complement of True alt ie. (90–True Alt). This is the third side of the triangle PZX, which has now become an arc of a great circle.

In the case of th Sun, HP is not separately tabulated in the almanac. Instead, the total correction tables already incorporate the parallax-in-alt. In case the parallax-in-alt for Sun has to be separately applied, it is calculated for various altitudes, and tabulated as a supplementary tables in the Norie's nautical tables, from which it can be directly taken and applied.

In the case of the planets Venus & Mars, the nautical almanac gives a supplementary correction tables adjacent to the total correction for stars. This is the Pax-in-alt correction to apply and is directly taken for the altitude concerned.

Back Angles : There may be instances, where it is not possible to observe the altitude of a body in the normal side of the horizon ie. on the side where the altitude is smaller. This can be due to the non-availability of the horizon on the normal side, either due to being hazy or foggy or due to nearness of land. In such cases, it may be necessary to take an altitude on the reverse side, ie. on the back side of the observer. Such observations are called "Back Angles", and all such altitudes are bound to be in excess of 90°.

There are two ways of correcting these observations to True Altitude. These are :-

1) Keep the altitude in excess of 90° right through. If so, all corrections are applied exactly same as a normal observation except that refraction and parallax are applied with opposite sign ie. refraction will be +ve & Pax will be –ve. However to obtain refraction from tables and to calculate pax-in-alt, it will be necessary to subtract the apparent alt. from 180° to obtain an approximate altitude on the normal side.

2) Use the supplement of the alt. observed for correction.

In this case after allowing for index error and dip as usual, find the supplement of the Apparent altitude and apply the remaing corrections in the normal way, except S.D., for which the signs are reversed. ie. LL. subtract, UL. add. It may be noted that sextant is capable of measuring angles only upto 120° approx.) Hence the minimum altitude of the body should be about 60° in the normal side, in order that the back angle can be measured.

Altitudes measured by Artificial Horizon :-

The equipments of an Artificial horizon consists of a jar containing mercury, a shallow metal trough, a glass shield whose glass is tested to be free from deformities, a metal spatula to clear the scum from the surface of mercury, if necessary. Mercury surface is used as the artificial horizion. Mercury is chosen, because it has a good shining surface which can reflect light very well, and secondly being a liquid, its surface will remain horizontal.

The artificial Horizon is set up by placing the metal trough on the ground or deck, on a stool or on a table. The mercury from the jar is poured into the trough. If there is any scum on the surface of mercury, it is removed by gently stroking the surface by the edge of the spatula. The surface is then covered by the glass

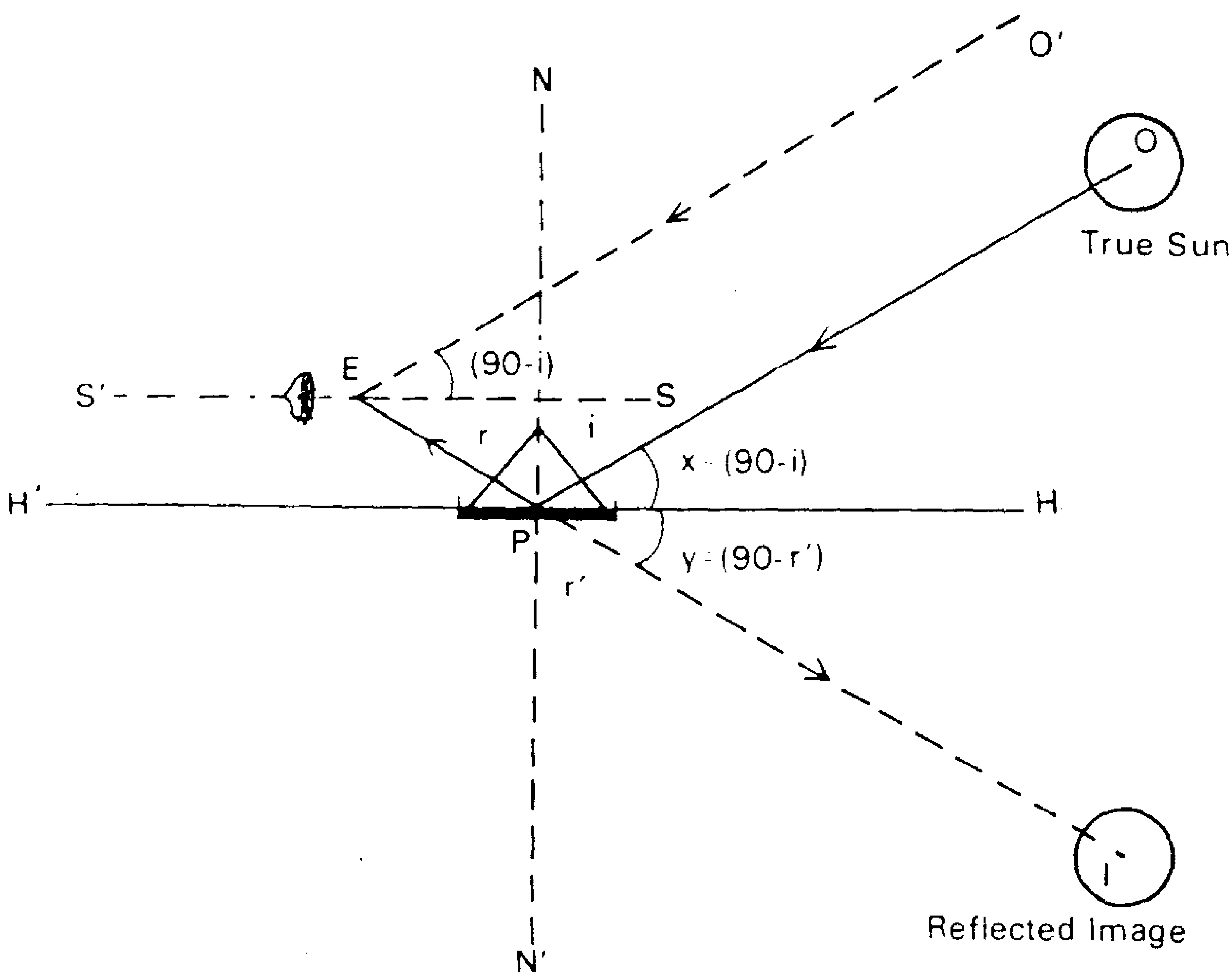


Fig. 21

In figure :-

HH' = Artificial Horizon (Surface of mercury).
NN' = Normal to the surface at Point 'P'.
SS' = Sensible Horizon Parallel to HH'.
E = Eye of the observer.
$O\hat{P}N$ = Angle of Incidence.
$N\hat{P}E$ = Angle of Reflection.
$O\hat{P}I$ = Measured Angle.
$O\hat{P}H$ = Apparent Altitude.

shield, to protect the surface of mercury from outside influence such as wind etc., so that the surface will remain undisturbed. Fig. 21 shows the arrangement.

A ray of light from the object O, (true sun in this case) strikes the surface of mercury in the trough at a point 'P' and is reflected to the eye 'E'. The eye perceives the Image of the sun inside the mercury at a point 'I'. With a sextant, the observer brings the reflected sun in coincidence with the reflected image of the sun, inside the mercury. He thus measures the angle OPI with his sextant.

Since the body is so far away, the ray striking the surface of mercury (OP) and the ray striking the eye direct from the body (O'E) are parallel rays.

To show the measured angle is twice the Apparent altitude.

Let the angle $O\hat{P}H$ be $x°$, and $H\hat{P}I$ be $y°$

$x = i = 90°$ ... Eq. I
$i = r$ (Laws of light)
$r = r'$ (vertically opp. angles)
$\therefore i = r'$
$r = y = 90°$ .... Eq. II

Comparing Eq. I & II :- we have
$x = y$
But angle x = Apparent altitude.
$\therefore$ Measured angle = $2x$ ($\because x = y$)
$\therefore$ Measured angle = Twice apparent altitude.

(Proved).

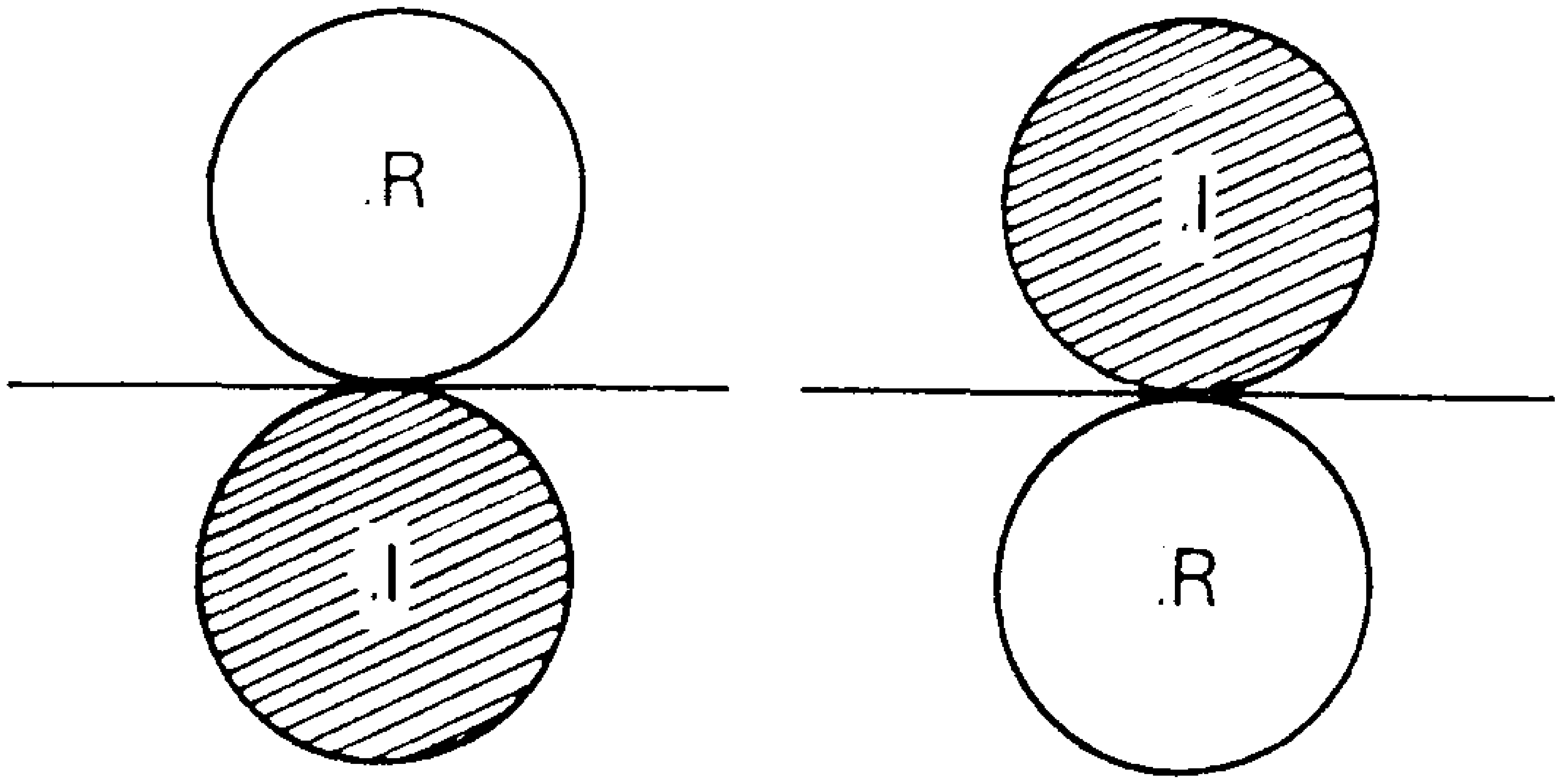
Correction of Altitudes taken on an Artificial Horizon (Ref. fig. 20).

Sextant Alt	$= O\hat{P}I$
I. Error	= (+ Off the arc and – on the arc)
Obs. Alt.	$= O\hat{P}I$ (assuming I. E. nil)
Apparent Alt.	$= O\hat{P}H = O\hat{P}I \div 2$
Apparent Alt.	$= O\hat{P}H$
Refraction	= –ve
S.D.	= (+ LL;– UL) *(optional)*
Parallax	= + ve
True Alt.	= – – –

Once apparent altitude is obtained the remaining corrections viz. refraction, S.D. and parallax are exactly same as for altitude taken on the visible horizon. Note that here is no correction to apply for Dip because as stated earlier rays striking the eye & the surface of mercury are parallel and hence $O\hat{P}H$ is also the angle $O\hat{E}S$ at the eye, which is the apparent altitude.

Another point to note is that the S.D. is an optional correction.

Where bodies do not have a S.D. as stars & Planets, the S.D. is not applicable in any case. But when using an artificial horizon even, an altitude of Sun or Moon, which present visible discs, need not necessarily qualify for the S.D. correction. S.D. correction need not be applied if the reflection of the body brought down by the sextant is made to coincide exactly with the image seen inside mercury. Then we are automatically making the centre of both reflections coinside and any S.D. correction is ruled out. This is only possible if the sextant is fitted with two distinct coloured shades, so that the exact overlap of one over the other can be distinguished clearly. If the sextant does not have such distinct coloured shades, then it is necessary to make the LL. of the reflection coincide with the upper limb of the image. In such a case, we are in fact measuring the altitude of one limb only and S.D. correction becomes applicable. This is illustrated in figure 22.



L.L. OBSERVATION U.L. OBSERVATION

Fig. 22

R reps reflection brought down by Sextant.
I reps Image seen inside mercury.

It may be observed that the centres of R & I are one diameter apart and yet we only apply S.D. correction. This is because of the fact, that by dividing the observed alt. by two we are automatically overlapping R & I by half its diameter. Hence S.D. correction alone need be applied.

Due to the limitation of sextant, which can only measure angles upto 120° and remembering the fact, that measured angle is twice the app. Alt. it is evident that if any altitude exceeding 60° (approx.) cannot be measured using an artificial horizon. Hence maximum altitude which can be measured using an artificial horizon is only about 60°

Though we have discussed correction of altitudes using an Artificial Horizon at length, its usefulness at sea is practically nil. Hardly any ships, nowadays carry the Artificial Horizon equipment. Secondly even if some older ships do, it is extremely difficult to obtain an undisturbed surface of mercury due to inherent vibrations on ships, due to engine and generator operations. Hence the discussion on Artificial Horizon is purely academical. It may be of some interest on shore based institutions.

Excercise V

(1) Define Visible, Sensible & Rational Horizon.

(2) What is "Augmentation to Moon's S.D." How and why is it applied ?

(3) What is "refraction correction"? Why is this normally negative. ?

(4) What factors affect parallax-in-alt. ?

(5) What is "Reduction" as applied to H P of moon & why is it applied ?

(6) Show : Parallax-in-alt. = H P x Cos app. alt.

(7) What corrections will you apply to a "Back Angle" observation ?
Why are some of the corrections reversed in this case ?

(8) When using an A.H., show that the measured angle is twice the apparent altitude.

(9) What are the limitations, when observing a back angle and an altitude using an A.H. ?

CHAPTER VI

NAUTICAL ASTRONOMY

Celestial sphere : The vast expanse of space that surounds the earth is called the celestial sphere. It has no limit and goes on endlessly. It appears to surround the earth like a vast dome placed over the earth. Since it has no limit, any point can be taken as the centre of this sphere. For this reason, it is convenient to assume that the celestial sphere is concentric with the centre of the earth and that all astronomical phenomenon that take place on the sphere, occur concentric with the centre of the earth. This assumption helps to understand all phenomenon without any error for practical purposes.

Celestial co-ordinates : Very similar to the terrestrial co-ordinates viz. latitudes and longitudes we have also celestial co-ordinates to express the position of any heavenly body. All points and co-ordinates on the earth, can be projected outwards to the celestial sphere and are given distinctive names to distinguish it from similar co-ordinates on the earth's surface.

Definitions :

Equinoctial : is a great circle, on the celestial sphere in the same plane as the equator on the earth. In other words, the equator when projected upwards & drawn on the celestial sphere is called the "Equinoctial"

Celestial pole : Earth's poles when projected on to the celestial sphere is called the celestial pole, and is 90° removed from the Equinoctial, just as the earth's poles are 90° away from the equator.

Parallels of Declination : Parallels latitudes on the earth, projected on to the celestial sphere are called parallels of Declination.

Declination : in the celestial sphere corresponds exactly to latitudes on the earth's surface, and helps to express the position of a heavenly body North or South of the Equinoctial.

Ecliptic : is a great circle on the clestial sphere drawn in the same plane as the earth's orbit round the Sun. Though the earth's orbit is an ellipse, the ecliptic as drawn on the celestial sphere is a great circle, in the same plane as the orbit. It is also to be noted that on the earth's surface, there is no such thing as Ecliptic, eventhough a Great circle can be drawn on the surface of the earth in the same plane as the Ecliptic, to represent the yearly apparent path of the sun round the earth. This line has no specific name on the earth.

Pole of the Ecliptic : A point 90° removed from any great circle is called its pole. Similarly Ecliptic, being, a great circle, has its own pole 90° removed from the ecliptic. This point is known as the pole of the ecliptic. This point also does not appear on the earth's surface, since, ecliptic itself does not appear on the earth.

Obliquity of the ecliptic : The angle at which the ecliptic crosses the equinoctial is called the obliquity of the ecliptic. This value is 23° 27' approximately and varies very slightly by a few seconds of arc each year on either side of its mean value quoted above. But for all practical purposes its value is assumed to the be constant at 23° 27' of arc without producing any appreciable error.

The first point of Aries : ♈ is the imaginary fixed point on the celestial sphere where the ecliptic crosses the equinoctial when the sun is moving from South to North in Declination. (see fig 23)

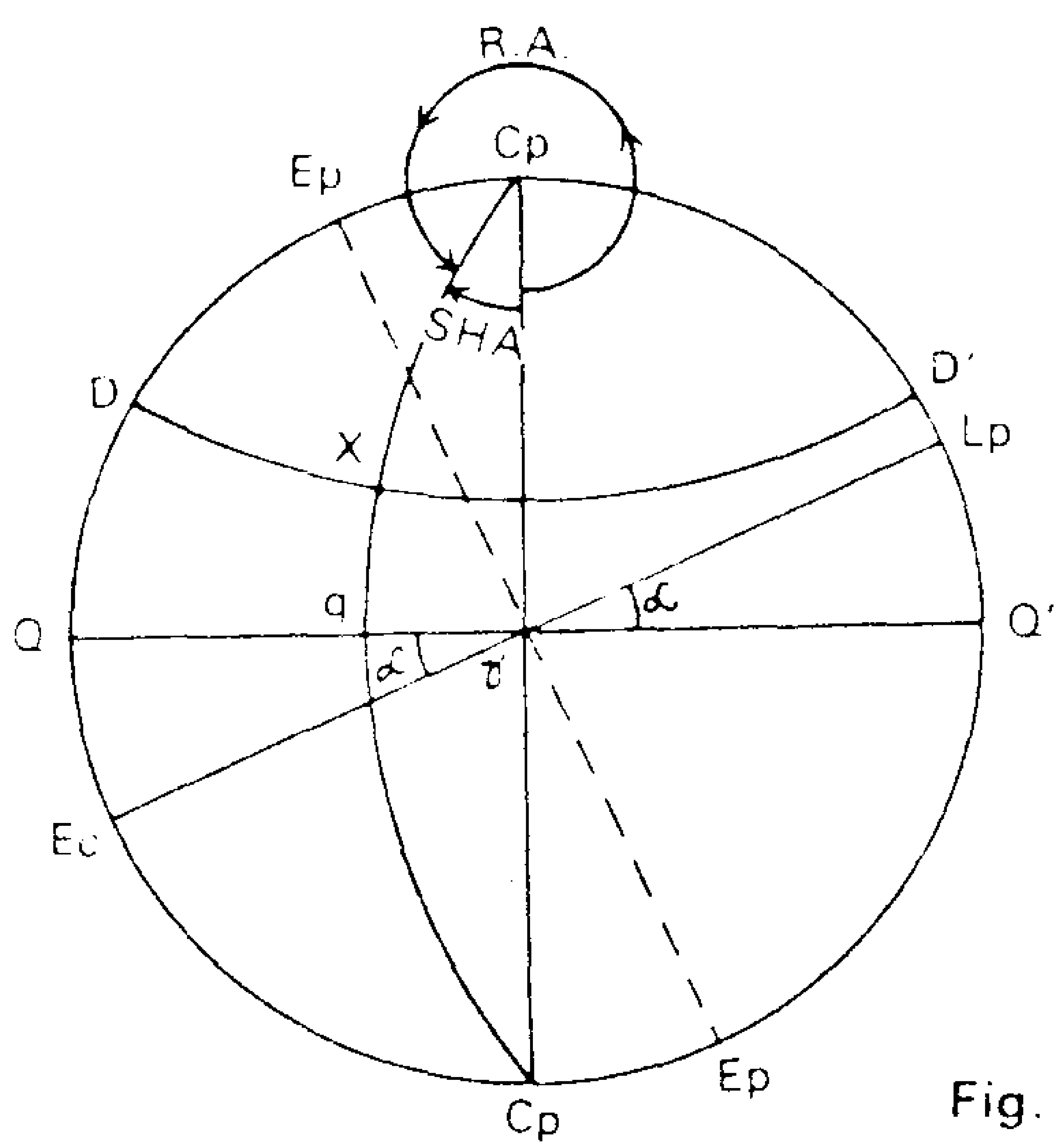


Fig. 23

In fig. 23

QQ	Reps	Equinoctial.
dd	Reps	Parallel of decl of X
Ec Lp	Reps	Ecliptic
Cp	Reps	Celestial pole
♈	Reps	1st point of Aries
O ♈ Ec	Reps	Lp $\hat{♈}$ O' = obliquity of the Ecliptic
Cp Cp	Reps	Clestial meridian
X	Reps	A Heavenly body
Arc qX	Reps	Declination of body X
♈PX	Reps	SHA of body X

First point of Libra : (♎) is the point where the Ecliptic crosses the equinoctial when the sun is moving from North to South Decliniation. This point is diametrically opposite to first point of Aries.

These are also the two fixed points of "Equinoxes" discussed in the last chapter, where the Decl of the sun is zero.

Celestial Meridians : Any meridian on the earth when projected upwards to the celestial sphere is a celestial meridian. In other words they are half great circles running from celestial pole to pole and crossing the equinoctial at right angles. (see fig. 23).

Of the infinite number of such meridians which can be drawn on the celestial sphere, that specific meridian passing through the first point of Aries has a special significance. It is a reference meridian from which the East/West co-ordinates on the clestial sphere are measured, something similar to the Greenwich (prime) meridian on the earth's surface from which longitudes east or west are measured.

Sideral Hour Angle : (SHA) is the angle at the clestial pole or the arc of the equinoctial contained between the meridian of 1st point of Aries and the celestial meridian passing through the heavenly body, always measured westewards and expressed in degrees, minutes and seconds of arc.(Arc ♈q or ♈$\hat{c}$pq in fig. 23).

Note that, unlike longitudes on the earth, which are measured both East & West of the prime meridian, SHA is always measured westwards only and will increase from zero degrees to 360°

First point of Aries being a fixed point in space, the Declination and SHA of heavenly body will give the fixed co-ordinates of that body at any time, and these are given in the nautical almanac for all heavenly bodies used for Navigational purposes. Thus by knowing the Declination and SHA of a heavenly body, that body can be pinpointed in the celestial sphere.

Right Assension : (R A) is the angle at the Celestial pole for or the arc of the equinoctial contained between the meridian of 1st point of Aries and the celestial meridian passing through the body, but always measured Eastwards and expressed in hours, minutes and seconds of time (see fig. 23).

Hence SHA + RA in arc = 360° always for anyone body. For purposes of navigation, however, the term RA is not commonly used.

Celestial Latitudes : are lines drawn parallel to the ecliptic both north & south of the ecliptic.

Celestial Longitudes : are similar to celestial meridians but drawn from North ecliptic pole to South ecliptic pole and crosses the Ecliptic at Right angles.

Both clestial Latitude & celestial Longitudes are parameters not used for the purposes of Navigation.

Navigational Application.

Since position fixing and navigation are done on the earth's surface, it becomes necessary to co-relate all what has been said earlier to the known co-ordinates on the earth's surface. Earth keeps spinning on its axis in West to East direction continuosly. Thus it appears to an observer on the earth, that all heavenly bodies rise in the east, reach a maximum altitude once a day when on the observer's meridian and later set in the west. Thus it makes a diurnal path across the sky each day.

Local Hour Angle : (LHA) (see fig. 24) is the angle at the elevated pole or the arc of the equinoctial contained between the observer's meridian and the meridian passing through the body at that time, measured westwards and expressed in degrees, minutes and seconds of arc.

LHA is always a Westerly Hour angle, unless it is specificially indicated as an Easterly Hour angle (EHA). Then it is an angle measured Eastwards from the observer's meridian to the meridian of the body. The necessity of this may sometimes arise, when the body is East of the meridian ie before its meridian passage.

LHA + EHA of a body = 360° (always)
ie EHA = 360 - LHA
or LHA = 360 - EHA

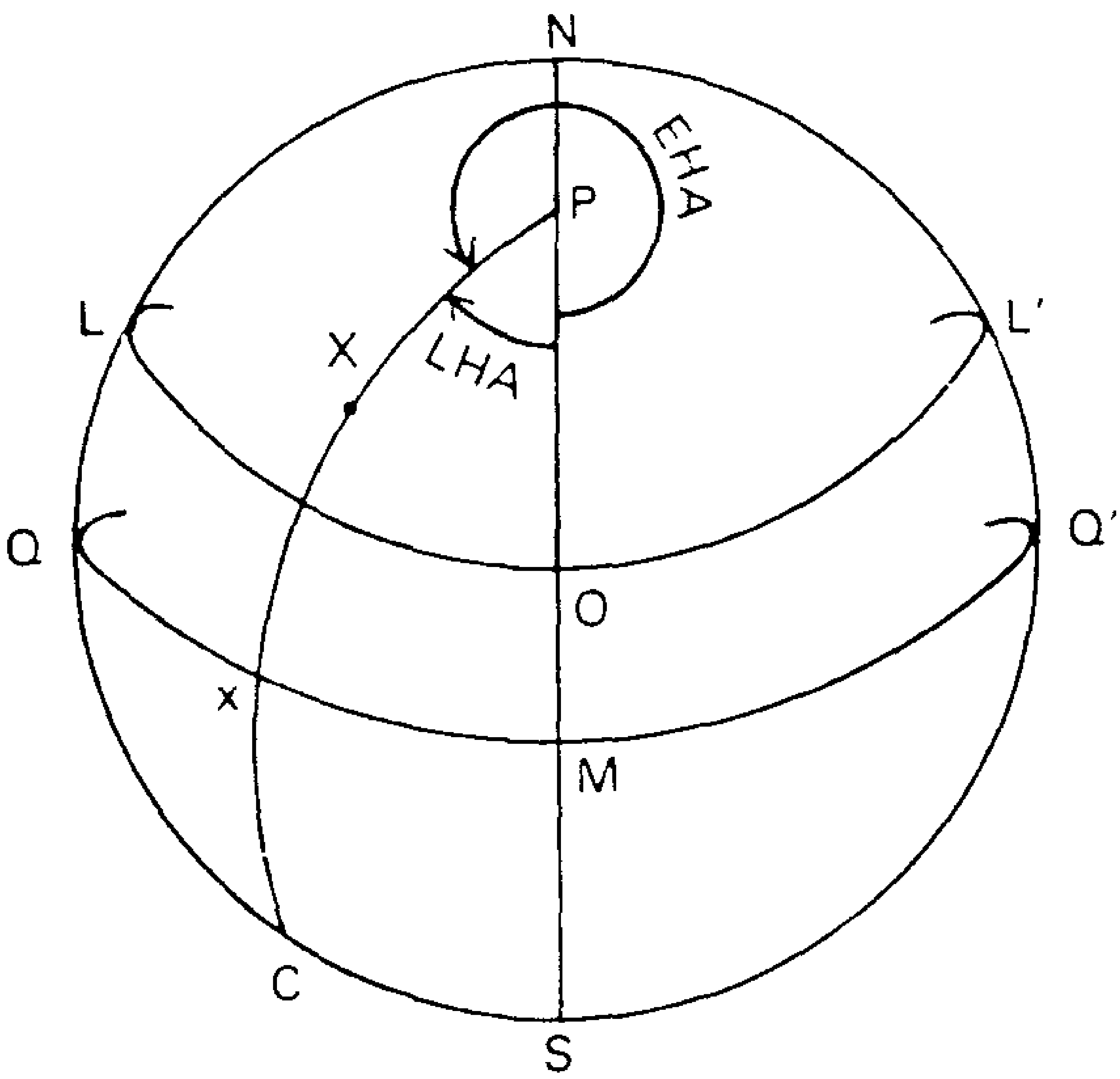


Fig. 24

In fig. 24
P - Elevated Pole.
QMQ' - Equinocital.
O - Observer
LL' - Latitude of observer
X-Heavenly Body
Arc Mx = $M\hat{P}X$ = LHA.
Reflex $M\hat{P}X$ = EHA.

Geographical position : of a heavenly body is that point on the earth's surface which is directly below that body at a given instant of time. In other words, it is that point on the surface of the earth, where a line drawn from the body to the centre of the earth cuts the crest of the earth. It is sometimes referred to as Sub-solar spot.

Likewise, the G.P. of a star is sometimes referred to as Sub-stellar spot.

If an observer is stationed at the G.P. of a body, that body will then be directly at his zenith.

Due to the rotation of the earth on its axis, a celestial body is continously changing its G.P. along its diurnal path each day. The G.P. will be a fixed point only for that instant at which the observation of the body is taken.

It is for this reason that it is not possible to adopt any fixed point on the earth as a reference point for fixing celestial co-ordinates for heavenly bodies, and we have to adopt fixed reference points only in space such as first point of Aries and the equinoctial; these being independent of the rotating earth's surface.

On the earth's surface, the position of an observer can be expressed in three different ways viz :

(1) By his latitude & longitude with reference to fixed co-ordinates on the earth.

(2) By a Bearing and distance from a fixed point of land.

(3) By a Bearing and the vertical Sextant angle of a tall object ashore, which will determine his distance off from the object.

Similarly the geographical position of a celestial body can be expressed in three different ways viz.

(1) By its Declination, corresponding to Lat on the earth and its Greenwich Hour Angle (GHA) at that instant of time.

(2) By its Bearing (Azmimth) and Zenith distance with reference to the observer.

(3) By its Bearing (Azimuth) with reference to observer and altitude of the body above the horizon.

The figure 25 is drawn on the plane of the observer's rational horizon on the principle of equidistant projection. This projection is the most commonly used projection for figure drawing in practically all navigational problems involving altitudes and azmiths of celestial bodies.

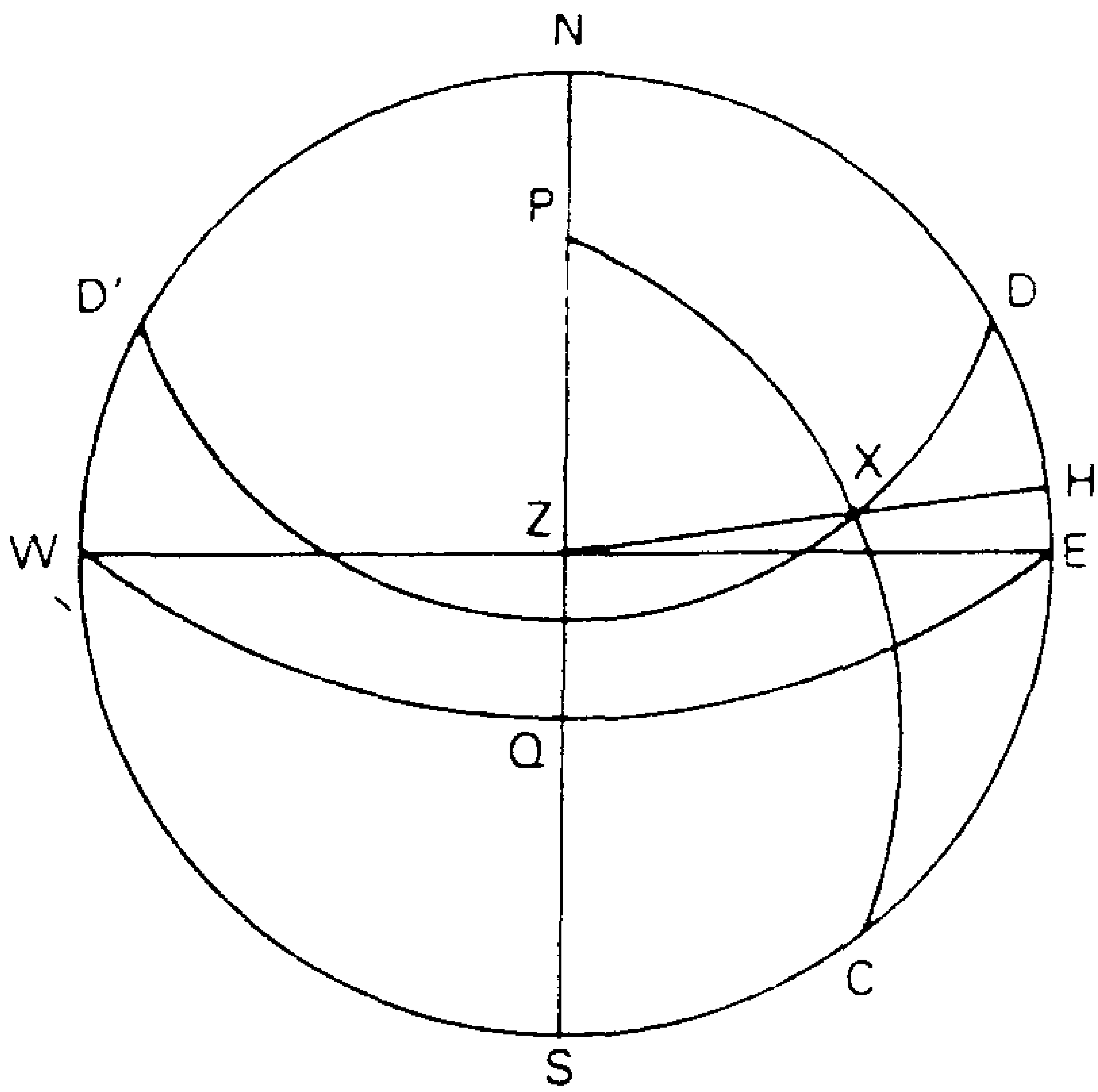


Fig. 25

Circle N E S W	reps.	Observer's Rational Horizon.
N Z S	reps.	Obs. Celestial meridian.
W Z E	reps.	Obs. Prime vertical.
W Q E	reps.	Equinoctial.
D X D'	reps.	Parallel of Dec. of the body.
P X C	reps	Celestial meridian of the body.
X	reps	Heavenly Body.
Z	reps	Obs. Zenith.
P	reps	Elevated Pole.
$Z\hat{P}X$	reps	Eastely Hour Angle.
$P\hat{Z}X$	reps	Azimuth.
Z X	reps	Zenith distance.
H X	reps	True Altitude of X.

Zenith : The point in the celestial sphere which is directly above the observer is called his Zenith.

Nadir : The point diametrically oposite to Zenith.

Vartical circle : is a great circle on the Celestial sphere which passes through the Zenith and cuts the rational horizon at right angles, hence called the vertical circle.

Prime vertical : is that specific vertical circle that passes through the East and West points of the rational horizon and crosses the observer's celestial meridian at the Zenith at right angles.

Azimuth : is the angle at the Zenith or the arc of the rational horizon contained between the observer's celestial meridian and the vertical circle passing through a specified celestial body at a specified time. It can be expressed in quadrantal notation of the compass or in three figure notion measured clock-wise from North ($P\hat{Z}X$ or the shorter arc N H in fig. 25). In short, it is the bearing of the body or its G.P. at a specified time.

Principle of position lines :

In fixing the position of an observer on the surface of the earth, by observation of celestial bodies, we use the G.P. of that body at a given instant of time, we do this, however, quite mechanically not quite realising, that, in fact, we are using the G.P. of that body, to determine the position. Hence G.P. of a body is an extremely important point for navigational purposes.

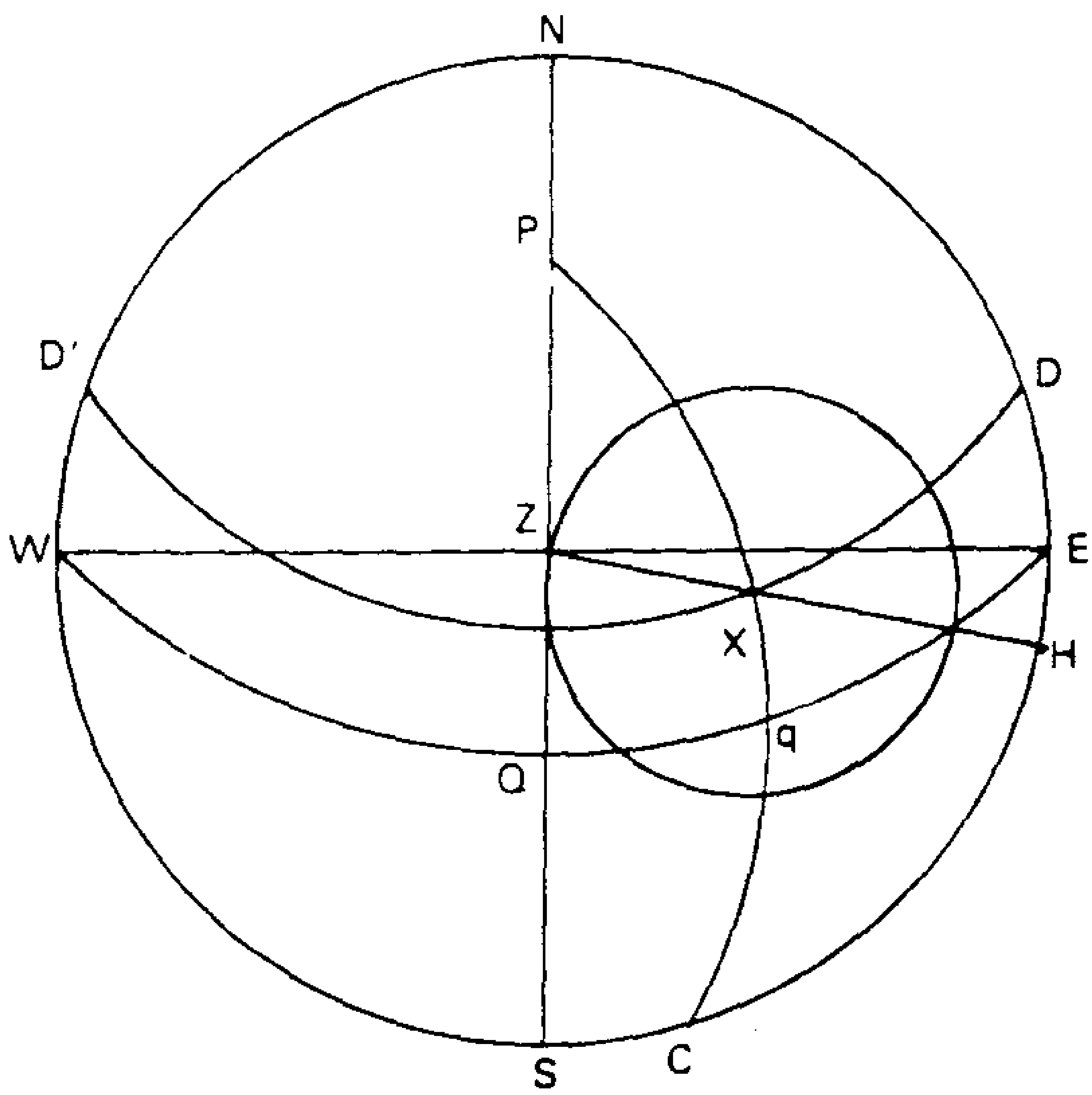


Fig. 26

All lines and points in the figure have the same meaning as are assigned in figure 25.

In all navigational problems, what we are mainly interested is solving the spherical triangle PZX where

PZ = Co-Lat ie (90 - Lat) QZ being latitude of obs
PX = Polar dist ie (90 - decl) qX being declination
ZX = Zenith dist. ie (90 - T, Alt) HX being True Alt

If we shrink this celestial triangle so that it may fall on the surface of the earth, point 'P' will represent the elevated pole, 'Z' will coincide with the observer and 'X' will coincide with the G.P. of the body at that time.

What we do know for certain at that instant of time is the G.P. of the body. The Lat and Long we use for working out a sight is only approximate ie it is only a D.R. position (Dead Reckoning position). We are trying to find out the actual position of the observer.

Referring to fig. 26 with G.P. of the heavenly body as centre, and the zenith distance (zx) as radius, if we draw a circle, the circumference of the circle will be very large on the earth's surface. If the observer is situated anywhere on the circumference, he will get exactly the same zenith distance. If at the same time, the bearing of the G.P. from the observer ie. the Azimuth of the body is also known, we can reverse this bearing from the G.P. and where it cuts the circumference of the circle will be the actual position of the observer. The circle, described above is called the **"Position circle"**. Though the circle is large, we are only interested in that part of the circle where the azimuth meets the circumference since the observer will be there only. The zenith distance (zx), being the radius of the circle, drawn in the direction of reversed azimuth from G.P. will meet the circumference at Z at right angles. On the earth's surface, it is most often not practical to draw such a huge circle, nor it is likely to fit within the limited area of the chart, the observer is navigating on. The G.P. itself may quite often be in entirely a different ocean or on a continent.

Since we are only interested in a small part of the circle, near about the observer's position, we only utilise that part of the circle. This tiny part of the huge circle can for all practical purposes be treated as a straight line at right angles to the azimuth without any error.

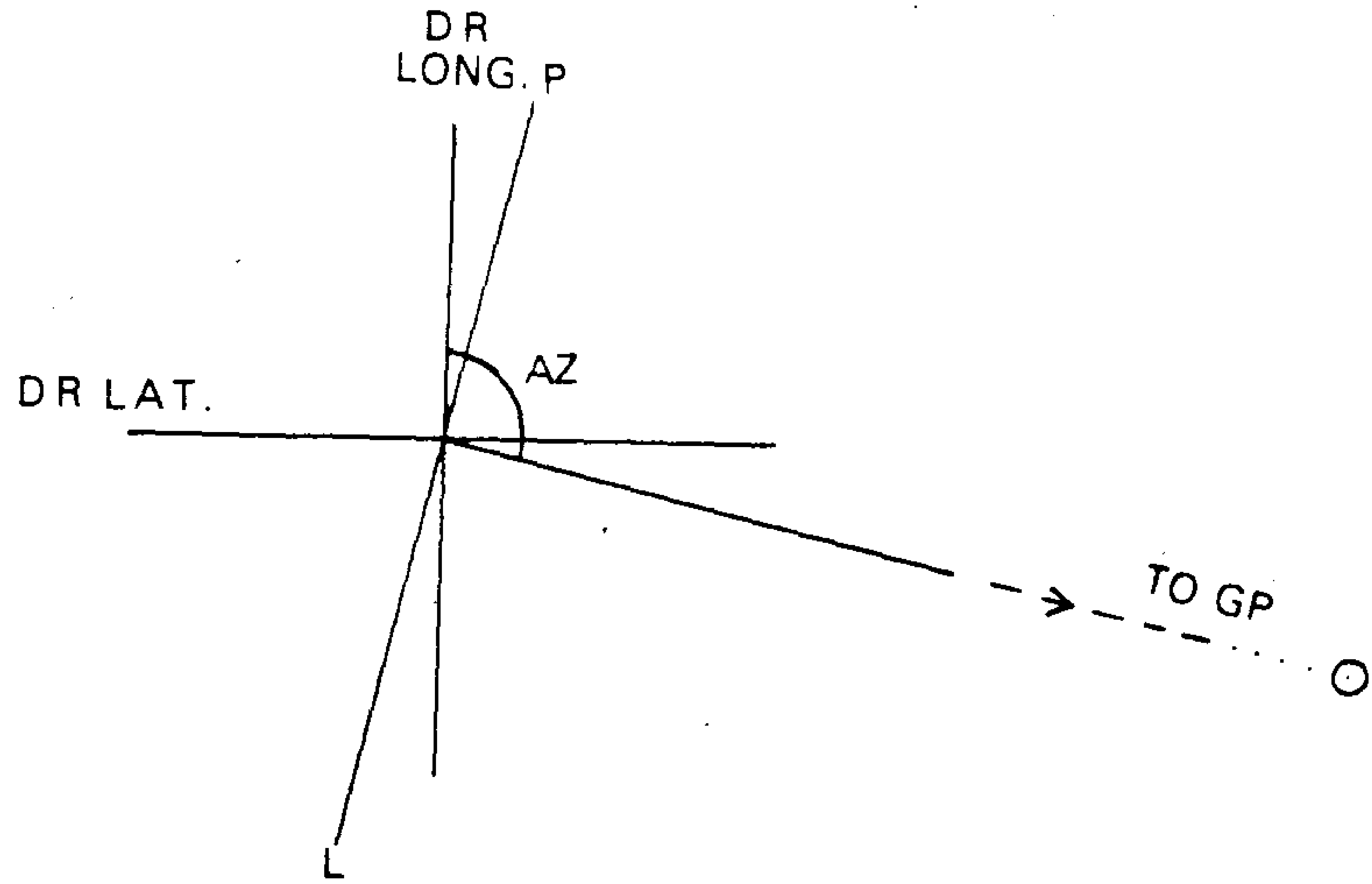


Fig. 27

In fact, that is what we do in practice. We draw a line at right angles to the Azimuth. This line is called a "**Position line**", which is a tiny part of the huge position circle. The observer need not necessarily be where the D.R. Lat. and Long. cross. The observer (Z) can be anywhere on this position line, as a small displacement of 'z' along ~~this line~~, will make no material plotable errors in the Azimuth or Bearing of G.P. which is thousands of miles away.

Thus in all novigational problems, one observation of a heavenly body will give only one position line (PL) on which the observer can be. Two such position lines will be required to know the exact position of the observer, which is determined by the meeting point of the two position lines.

Marcq st. Hilaire (Intercept) method of Position Lines :

Using a DR Lat. & Long. of the observer, the triangle PZX is solved.

PZ = 90 - Lat. = Co-Lat.
PX = 90 $\pm$ Decl. = Polar distance.
Z$\hat{P}$X = Hour Angle of body X.

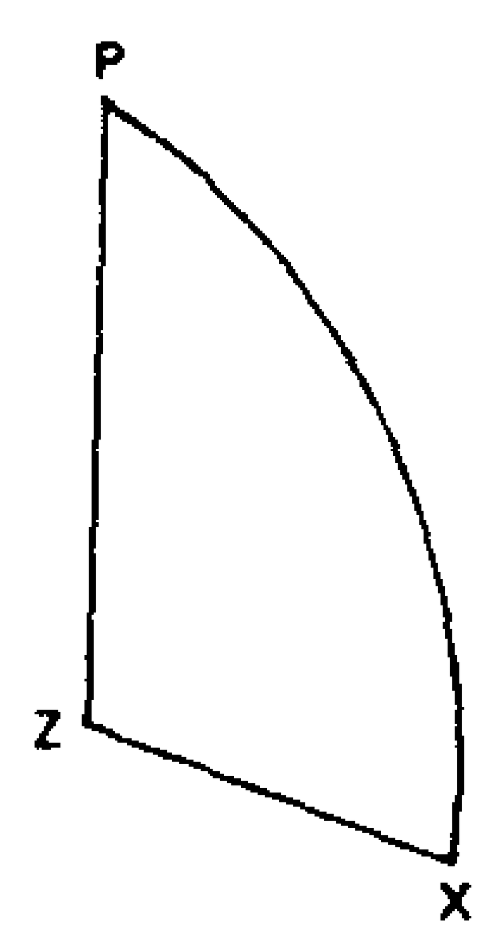


Fig. 28

These three parameters being known the ZX is calculated using the Haversine formula viz.

$$Hav\ ZX = Hav\ P\ \sec PZ\ \sec PX + Hav[PZ \sim PX]$$

Taking complements we can rewrite the formula as

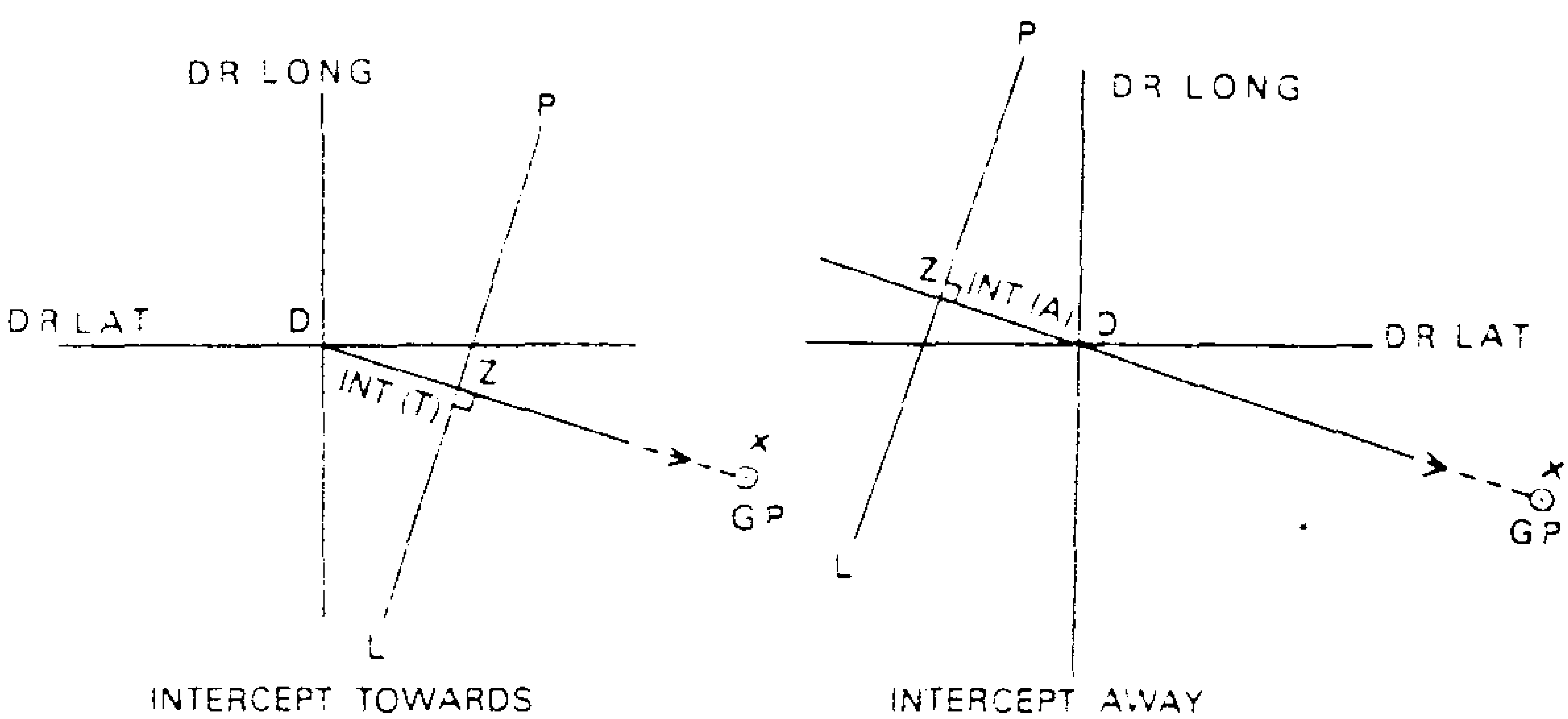
$$Hav\ ZX = Hav\ P\ Cos\ Lat.\ Cos\ Decl. + Hav[Lat \sim Decl.]$$

With this we get a calculated value for the Zenith distance assuming the observer to be at the DR Position.

We also calculate the $P\hat{Z}X$ ie. the Azimuth by 'ABC' Tables in Norie's or Burton's nautical tables.

Having taken the altitude of the heavenly body earlier and converting it into true altitude, we can find the True Zenith distance, which will be 90 – True Altitude.

We now compare the true Zenith distance with the calculated Zenith distance as follows :-



(A) Fig. 29 (B)

In figures 29 (A) & 29 (B)
= the G.P. of the body very far away.
X = the calculated Zenith distance using the DR.
X' the True Zenith distance as obtained by the sight

fig. 29 (A) ZX is shorter than DX By DZ.
fig. 29 (B) ZX is longer than DX by DZ.

This quantity DZ, is nothing but the difference between the Calculated Zenith Distance and the True Zenith Distance.

If as in fig. 29 (A) the Calculated Zenith Dist. exceeds True Zenith Distance, the Intercept is termed "**Towards**" but if the Calculated Zenith Distance is less than the True Zenith Distance as in fig. 29 (B) we term the intercept as "**Away**".

Point Z, called the Intercept Terminal Point (ITP), is the point through which to draw the position line at right angles to the azimuth. So the rule is :-

CZD > TZD Int. Towards.
CZD < TZD Int. Away.

Had we used a different D.R. lat. for the sight, the position line itself will not change, but the point through which to draw the PL will change. This is illustrated in fig. 30.

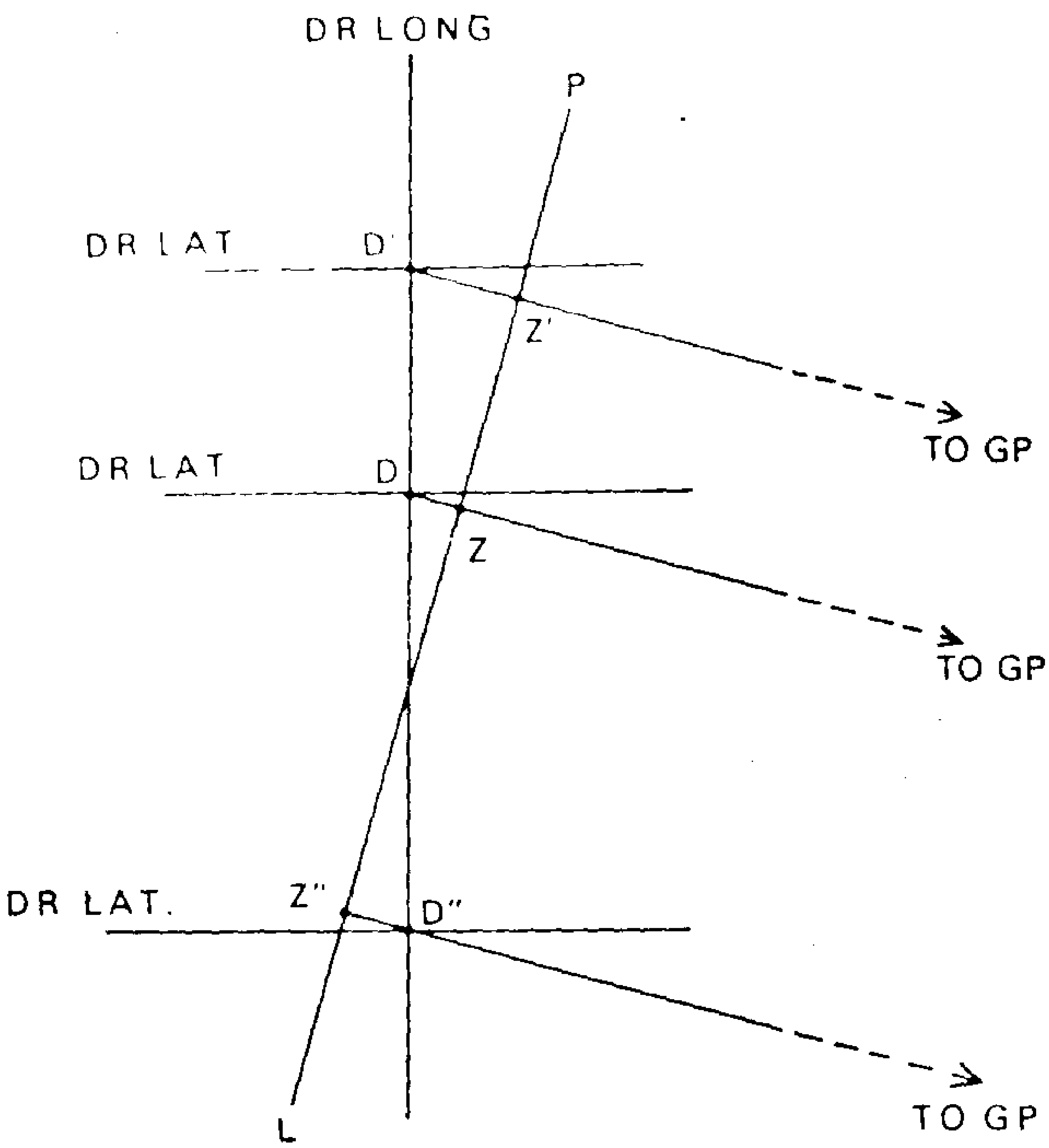


Fig. 30

As shown in fig. 29, instead of using D.R. Lat. of D, had we used D.R. Lat. of D′ we would have got D′Z′ as the intercept (towards). P L would have been drawn through Z′. Likewise, if we had used D.R. lat. of D″ for the sight, the intercept would have been D″Z″ (away) and PL would have been drawn through Z″. Notice in all these cases, the direction of Azimuth and the P L have not changed at all. What has changed is the quantum of Intercept and the position through which to draw the P L

Longitude Method of Position lines :

In solving triangle PZX by the longitude by chronometer method we calculate P, using the Haversine formula as stated below.

In △ PZX

Given PZ = Co-lat = 90 − Lat.

PX = Pol. Dist. = 90 ± Decl.

ZX = True Zenith dist. as obtained at sight.

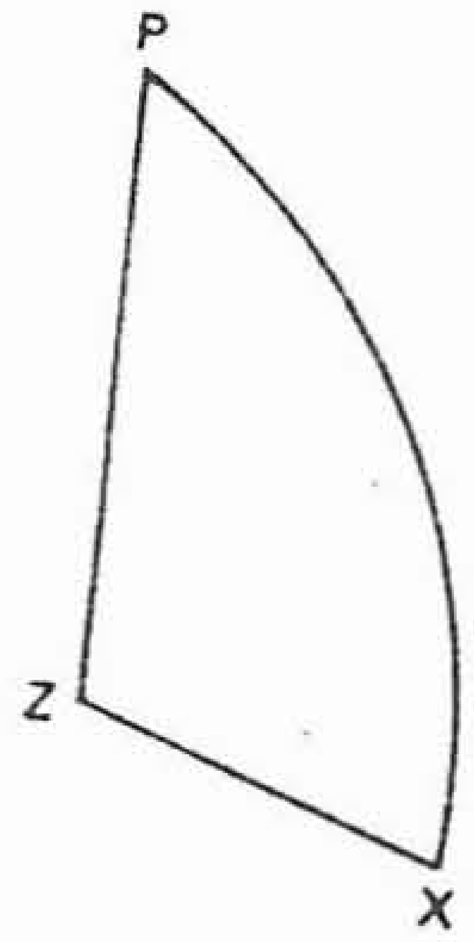


Fig. 31

Hav P =[Hav ZX - Hav (PZ~PX)]Cosec PZ Cosec PX

Taking complements, this formula can be rewritten as :

Hav P =[Hav ZX - Hav (Lat~Decl)] Sec Lat. Sec Decl.

Having got P : if it is EHA this has to be subtracted from 360° to make it LHA.

LHA = 360 − EHA.

For the GMT of observation, GHA of the body is obtained from the Nautical Almanac.

Longitude = GHA ~ LHA

GHA > LHA = Long is West

GHA < LHA = Long is East

The Azimuth is then calculated for the H.A. as found above, using ABC tables.

The PL is then drawn through a position at the intersection of the D.R. Latitude with the observed Longitude. It should be noted that this longitude so obtained is correct provided the DR Lat. used for the sight is correct. If we had used a different DR Lat. the observed longitude and the position to draw the P L would also change. This is illustrated in fig. 32

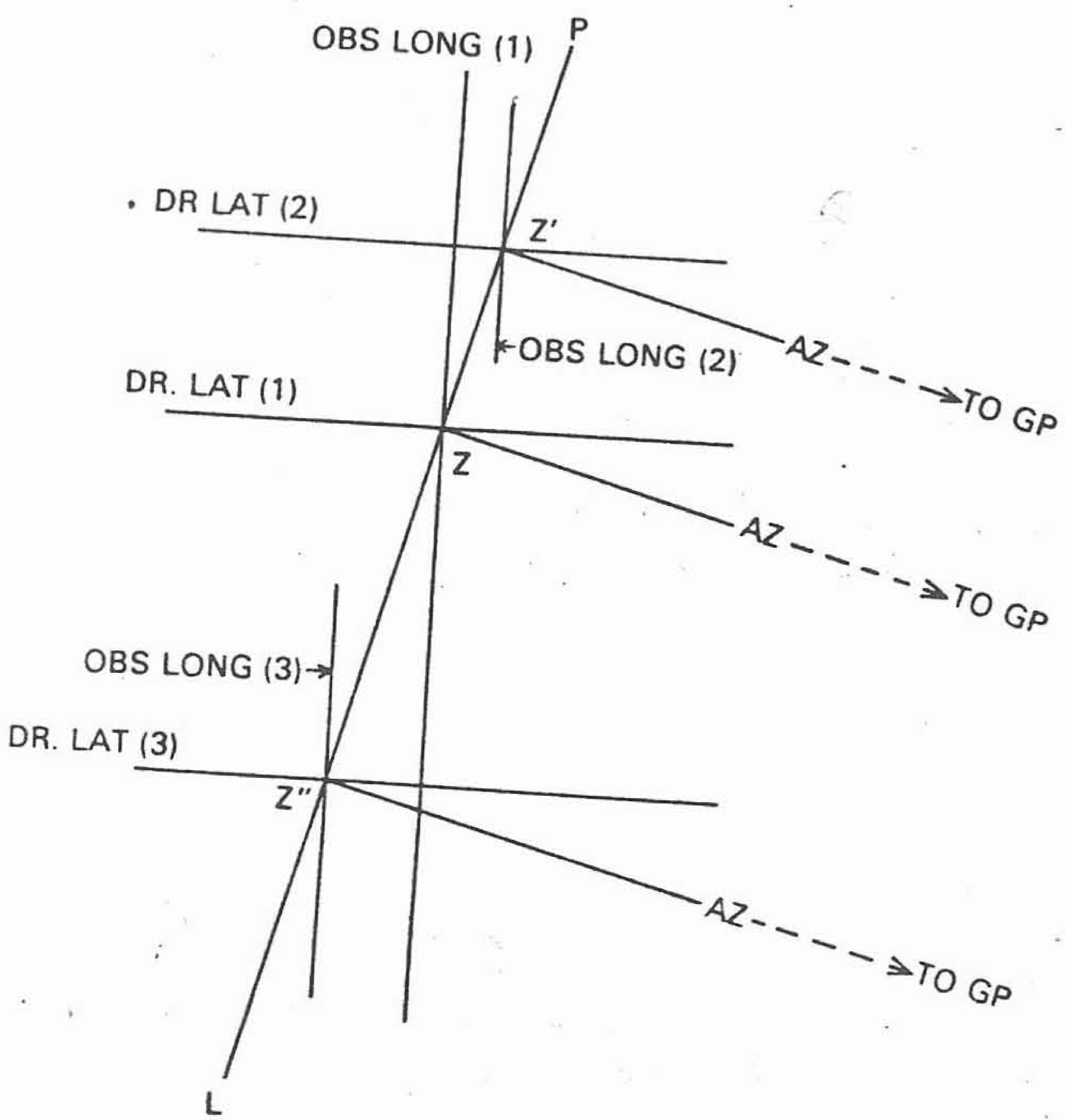


Fig. 32

Using DR Lat. (1) we get obs. Long. (1) and PL is drawn through Z, the meeting point of DR Lat. & obs. Long. Had we used DR Lat. (2), we would have got obs. Long. (2) & Z' being the position, through which the PL would then be drawn. Similarly, if we had used DR Lat. (3), we would get obs. Long. (3) and position for PL as Z". Notice in all these cases, provided, the DR, Lat. is within reasonable limits, the Azimuth & PL do not alter. What alters is the obs. Long. and the the position through which to draw the P L.

One interesting observation to note in Longitude problems is that, as long as the P L in running more or less N/S even an appreciable difference in DR Lat., makes very little difference in the observed longitude. This is very evident from figure 32. The PL can run more or less N/S only if the Azimuth is more or less E/W, since they are at right angle to each other. If the Azimuth is exactly East or West, the PL will exactly be N/S which will then coincide with the observed longitude itself. In all longitude by chronometer observations, this is the ideal we try to achieve, by taking the observation when the body bears, due East or West. This occurs if the body is on the observer's prime vertical. The ideal is not always achievable. So we try to time a long by chronometer observation at such a time, when the body is as close to the Prime vertical as possible, so as to get an Azimuth almost E/W so that PL will be almost N/S.

A body will be on the observer's prime vertical twice a day, once before meridian passage and once after meridian passage, provided, the lat. & Decl. are same names and latitude is numerically greater than declination.

Bodies whose azimuth is closer to N/S, will produce position lines running more E/W. In such cases, even a small error in DR Lat., will, produce an appreciable error in observed Longitude, and hence cannot be relied upon. Therefore bodies, closer to the meridian are not suitable for Longitude observations. Because of these limitations, one has to be rather selective in chosing bodies for longitude observations, whereas no such limitations exist for intercept method of P L.

Ex-Meridian Method of Position Lines :

When a body is close to the meridian within certain limits a position line & a position through which to draw the PL can be obtained using the Haversine formula or by using Ex-Meridian Tables given in any nautical tables.

Use of Haversine formula is as follows :-

When a body is close to the meridian, note HA (P) is very small (see fig. 33)

In $\triangle PZX$, given
PZ = 90−Lat. = Co−Lat.
PX = 90 ± Decl. = Pol. dist.
ZX = 90−T. Alt. = Zenith distance.

$$Hav\,(PZ \sim PX) = Hav\,ZX - Hav\,P\;Sin\,PZ\;Sin\,PX.$$

Taking complements, the formula can be rewritten as

$$Hav\,(Lat. \sim Decl.) = Hav\,ZX - Hav\,P\;Cos\,Lat.\;Cos\,Decl.$$

(Lat. ∼ Decl.) is the meridinal zenith distance of the body, very close to the meridian.

From this MZD, and knowing the declination, the Latitude can be computed.

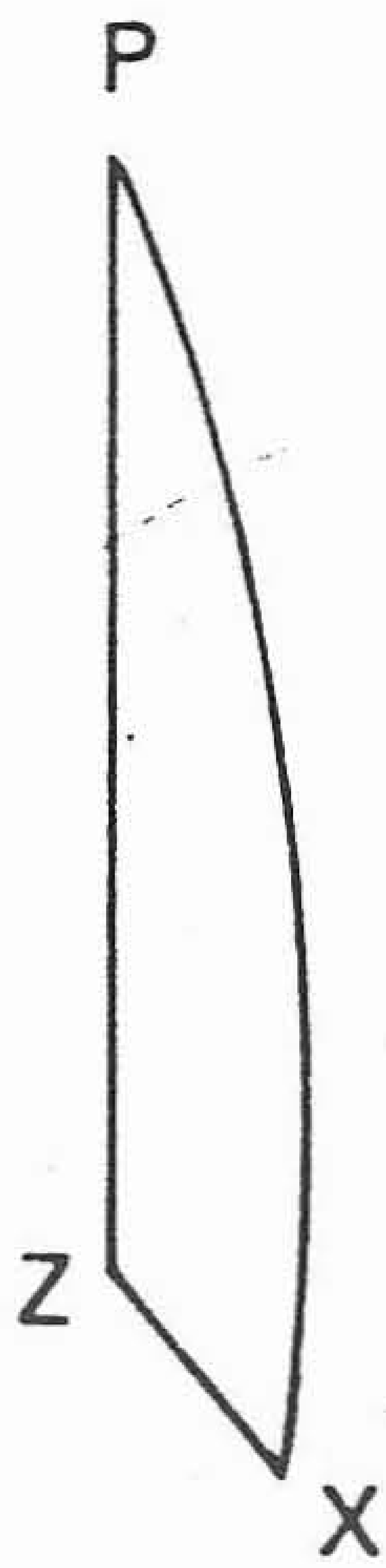


Fig. 33

Using the Latitude obtained, Declination & HA used for the sight. Azimuth, is worked out from ABC tables. PL is ± 90° to the Azimuth.

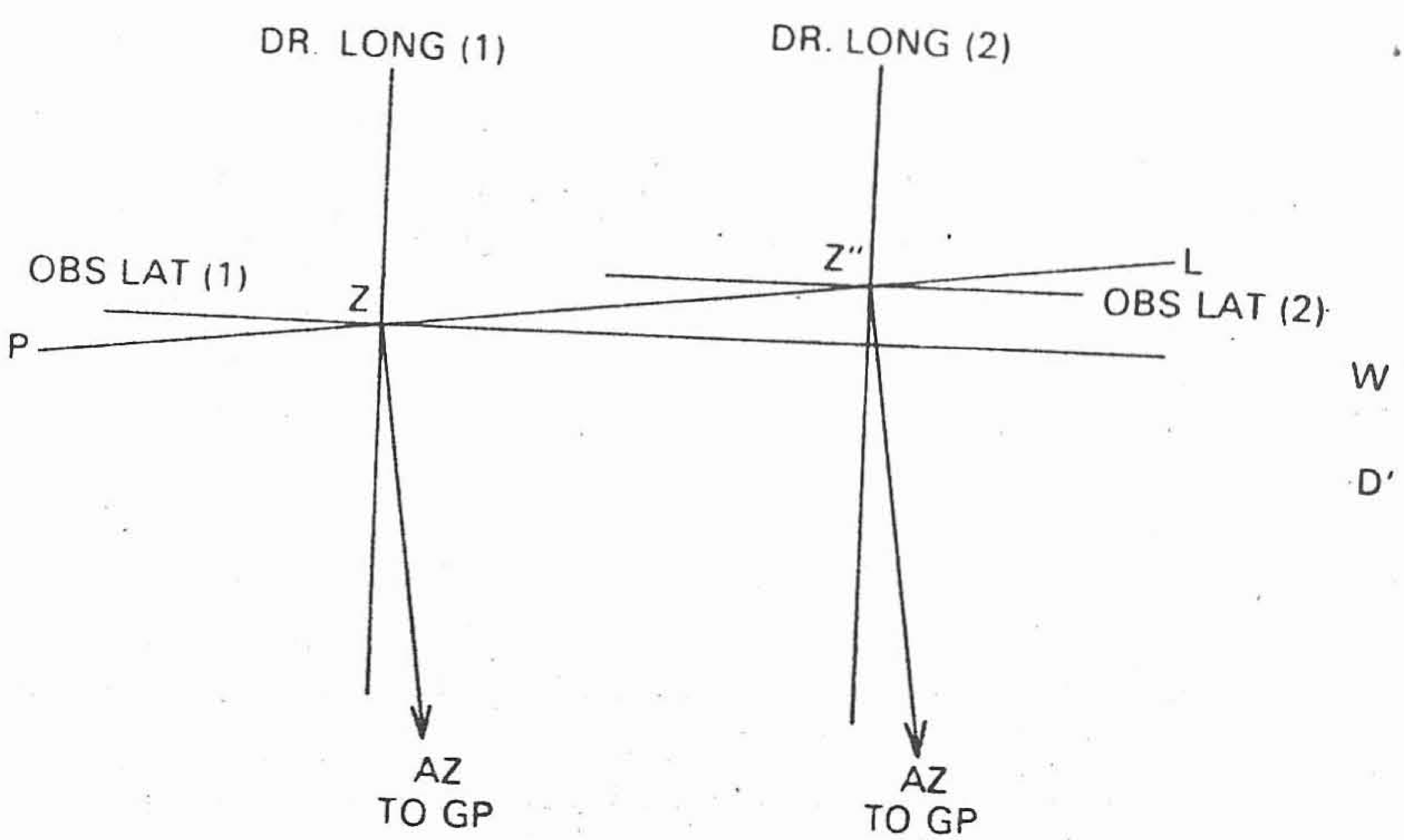


Fig. 34

The P.L. is drawn at right angles to the Azimuth through the intersection point of the observed Lat. and D R. Longitude as shown in fig. 34.

Note that the observed Latitude obtained is correct only if the D.R. Long. used for getting the HA is correct. Had we used DR Long. (2), then we would have got observed lat. (2) and the PL would be drawn through Z' instead of through Z. Notice again, the Azimuth & PL have not changed. What has changed is the obs. Lat and the position to draw the P.L. Notice also, the fact that, the Azimuth being nearly N/S the P.L. is running more or less E/W so that even an appreciable difference in DR Long. makes very little difference in the observed Latitude.

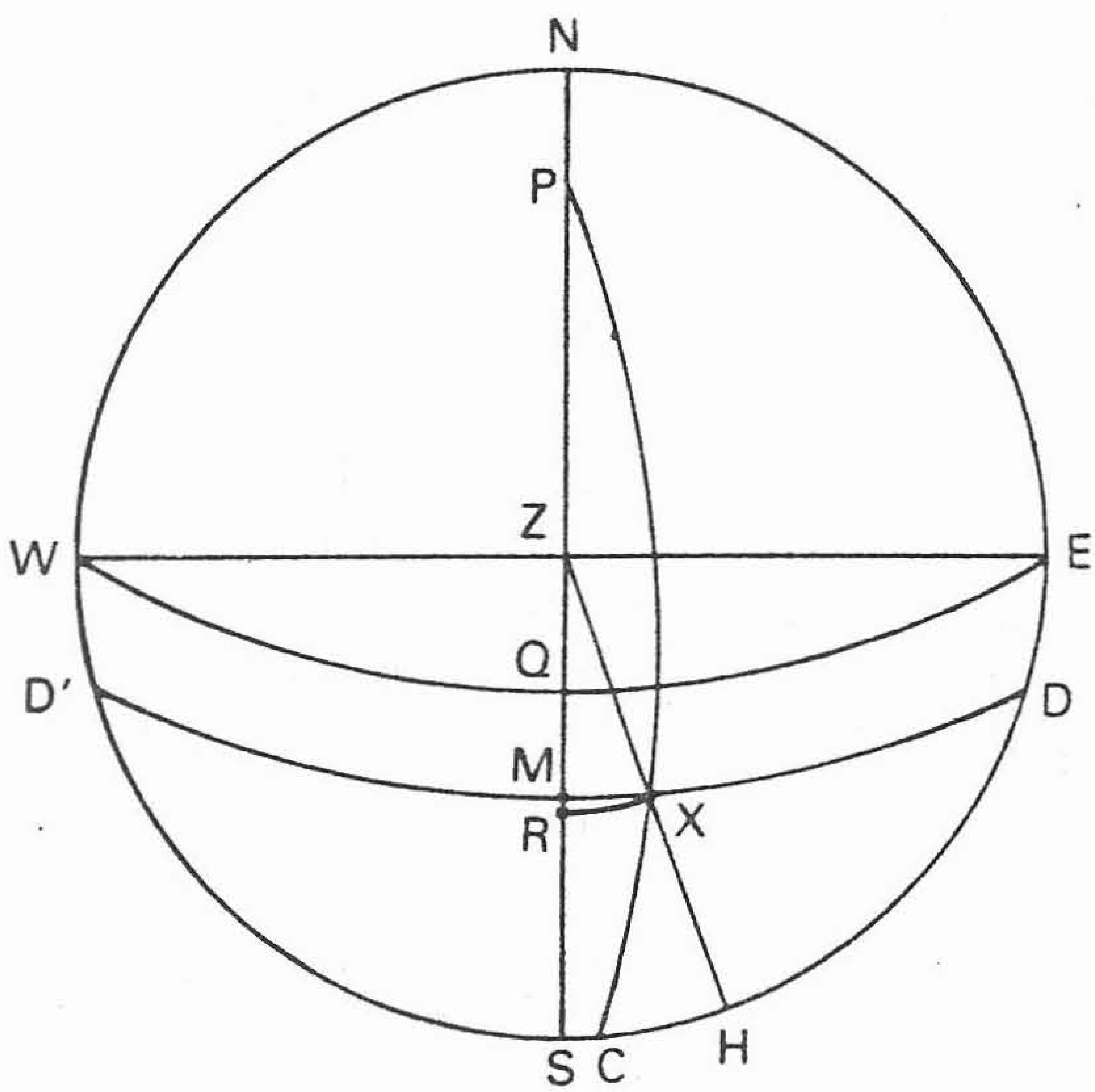


Fig. 35

Principle of Ex-Meridian Tables.

The explanation of the figure 35 is same as shown earlier and hence not repeated.

Construction: With Z as centre and ZX as radius cut off an arc on the meridian at 'R'.

$\therefore$ ZX = ZR

'M' represents the point where the body would be when it reaches the meridian & ZM will be the Meridinal Zenith distance. A small triangle MXR is obtained due to the aforesaid construction. Because the triangle is very small, we take certain liberties and treat the triangle some what like a plane triangle and calculate the value of "MR" by using Ex-meridian tables I, II & III.

ZR - MR = ZM (Meridinal Zenith Dist.)

ZM ~ Decl. = Latitude.

The calculated value of 'MR' is called the "**Reduction**" and is always subtracted from the Zenith distance to obtain Meridinal Zenith Distance. (MZD).

Ex-Meridian Table I is the rate of change of altitude per minute of time in the H.A.

Ex-Meridian Table II gives the computed change in altitude for the actual H.A. in use or in other words, it is the computed value of 'MR'.

Ex-Meridian Table III is a small supplementary correction to be applied to 'MR' for compensating for assumptions made in solving △ MXR.

In order that the calculated value of 'MR' is sufficiently accurate, it is necessary that the HA is small enough to make △ MXR small. Such limitation of the HA upto which a sight can be worked out as an Ex-Meridian sight is given under Ex-Meridian Table IV.

Having obtained latitude the procedure for Azimuth & PL etc. are as explained earlier.

Exception : The reason why a P L is at right angles to the Azimuth was discussed at length earlier. However there is an exception to this general rule. When the body is on the meridian it is either bearing due North or South of the observer. The P L being at right angles to the Azimuth, coincides with the latitude. Hence for all meridian altitude situations, the latitude itself is the P L except in one situation as illustrated in fig. 36.

Take the case of a body whose true altitude when on the meridian is very close to 90°; say 89°56' and let the Azimuth of the body be 180° when on the meridian. This means that the G.P. of the body is 4' of arc or 4 nautical miles due south of the observer. This is such a short distance that the G.P. can be plotted on the chart, the observer's meridian coinciding with the longitude of the G.P. of the body.

With G.P. of body is as centre and radius = M.Z.D. (4' in this case) a circle can be drawn on the chart. Reverse the bearing of the body from the G.P. to meet the circle at Z, which is now the observer's position. Notice that the PL is not a straight, line at right angles to Azimuth, but arc of circle, passing through Z (see fig. 36).

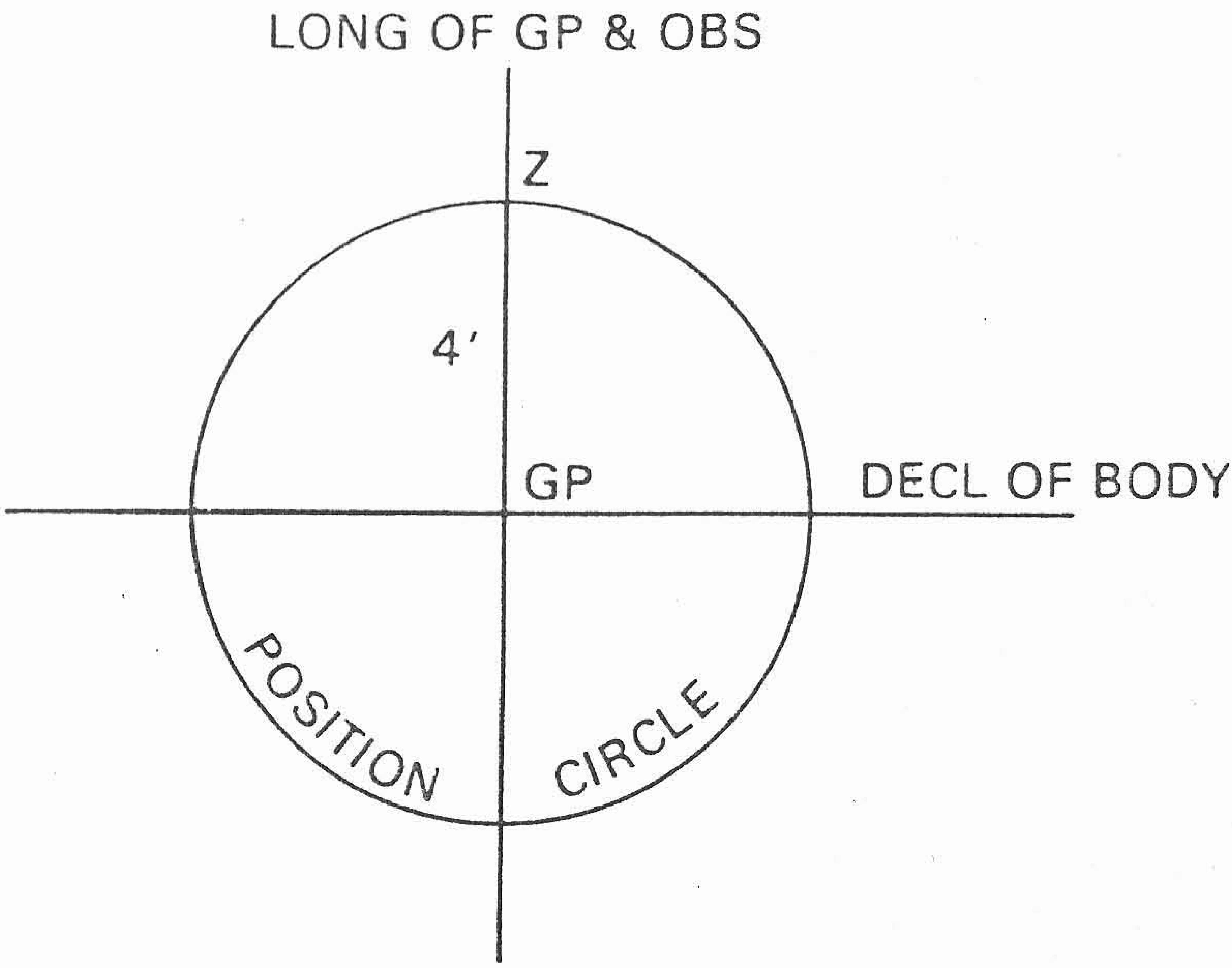


fig. 36

We can therefore, say, that, if the Zenith distance is small enough to be drawn on a chart, then it is not correct to say that PL is at right angles to the Azimuth in that specific case.

Time Amplitude

There is one Azimuth taken at rising & setting of a heavenly body which is called the Amplitude.

Definition : Amplitude is defined as the angle at the Zenith or the arc of the rational horizon contained between the observer's prime vertical, and the vertical circle passing through the body when the body is on the observer's rational horizon, ie. at rising or setting. In short it is the complement of Azimuth at rising and setting.

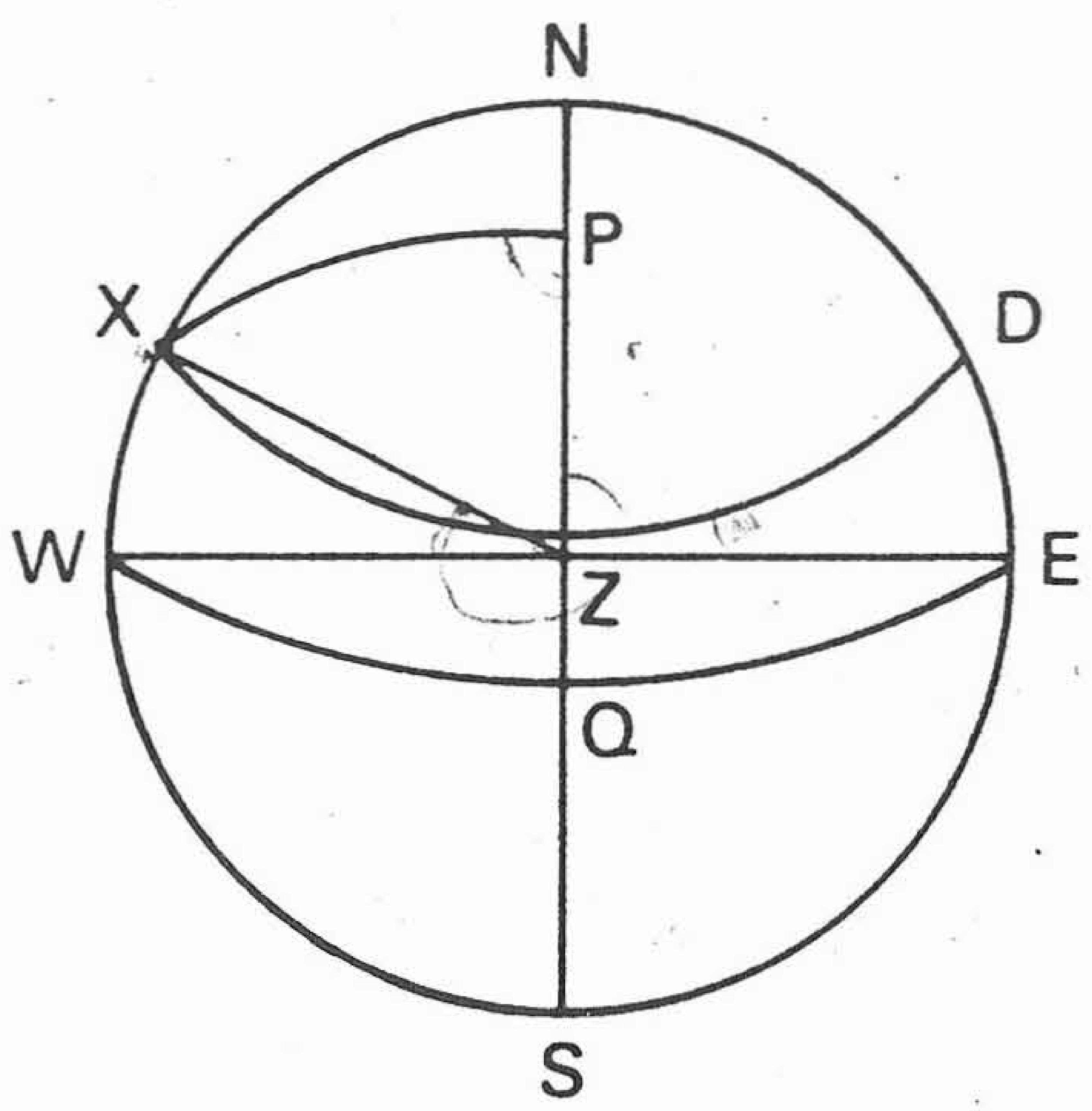


Fig. 37

Sin Amp = Sin Decl. Sec lat.

* In the quadrantal △ PXZ
PX = (90±decl.) = Polar distance.
PZ =(90 – Lat.) = Co-Lat.
ZX = 90°
To find $\hat{Z}$
Using the Napeir's Rules for circular parts.
Sin (90 – PX) = Cos $\hat{Z}$ Cos (90 – PZ)
Cos $\hat{Z}$ = Sin (90 – PX) Sec (90 – PZ)
Cos $\hat{Z}$ = Cos PX Cosec PZ
Taking complements.
Sin Amp. = Sin decl. Sec Lat.

(Proved)

The only two heavenly bodies which can be used for amplitude is Sun & Moon.

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* Proof of this will be better understood after studying quadrantal triangles in chapter XIII.

The correct time to take the amplitude of the sun is when its LL is half its diameter above the horizon, because only at that time its true altitude is zero. This is because of refraction being as much as 32′ of arc at zero degrees altitude which is practically equal to the diameter of the sun itself.

Excercise VI

(1) Define the following terms :
Declination, Ecliptic, Obliquity of the Ecliptic, Prime vertical, First point of Aries, SHA, LHA, Geographical position, Azimuth, Amplitude.

(2) Explain why it is necessary to use fixed points in space as reference points for expressing celestial co-ordinates ?

(3) Express three different ways in which, a terrestrial position and a geographical position of a heavenly body can be indicated.

(4) What is a position circle and how is it obtained ?

(5) What is a position line and why is it at right angles to the Azimuth ?

(6) Under what circumstances will a position line not be at right angle to the Azimuth and why ?

(7) What is the most suitable time to observe a body for longitude by chronometer and why ?

(8) Prove the formula : Sin Amp. = Sin Decl. Sec Lat.

(9) What is the best time to obtain the amplitude of the sun and why ?

CHAPTER VII

SOLAR TIME

For time immemorial people on the earth, have had some means to record events and phenomenon in their chronological order. This is how a layman would define time as. For purposes of astronomy and navigation we go much more into details.

For civil purposes, we use the true sun as the time keeper, because of the advantage the sun bestow on us in dividing the day into day & night.

The unit of measurment of time is, a **Solar day** which is defined as the interval between two successive transits (meridian passage) of the Sun over the same meridian. Since this is too large a unit for practical usage, it is further subdivided into hours, minutes and seconds.

Apparent Solar day : is the interval between two successive transits of the True Sun (ie the Sun we see) over the same meridian. Its length is not constant for the following reasons.

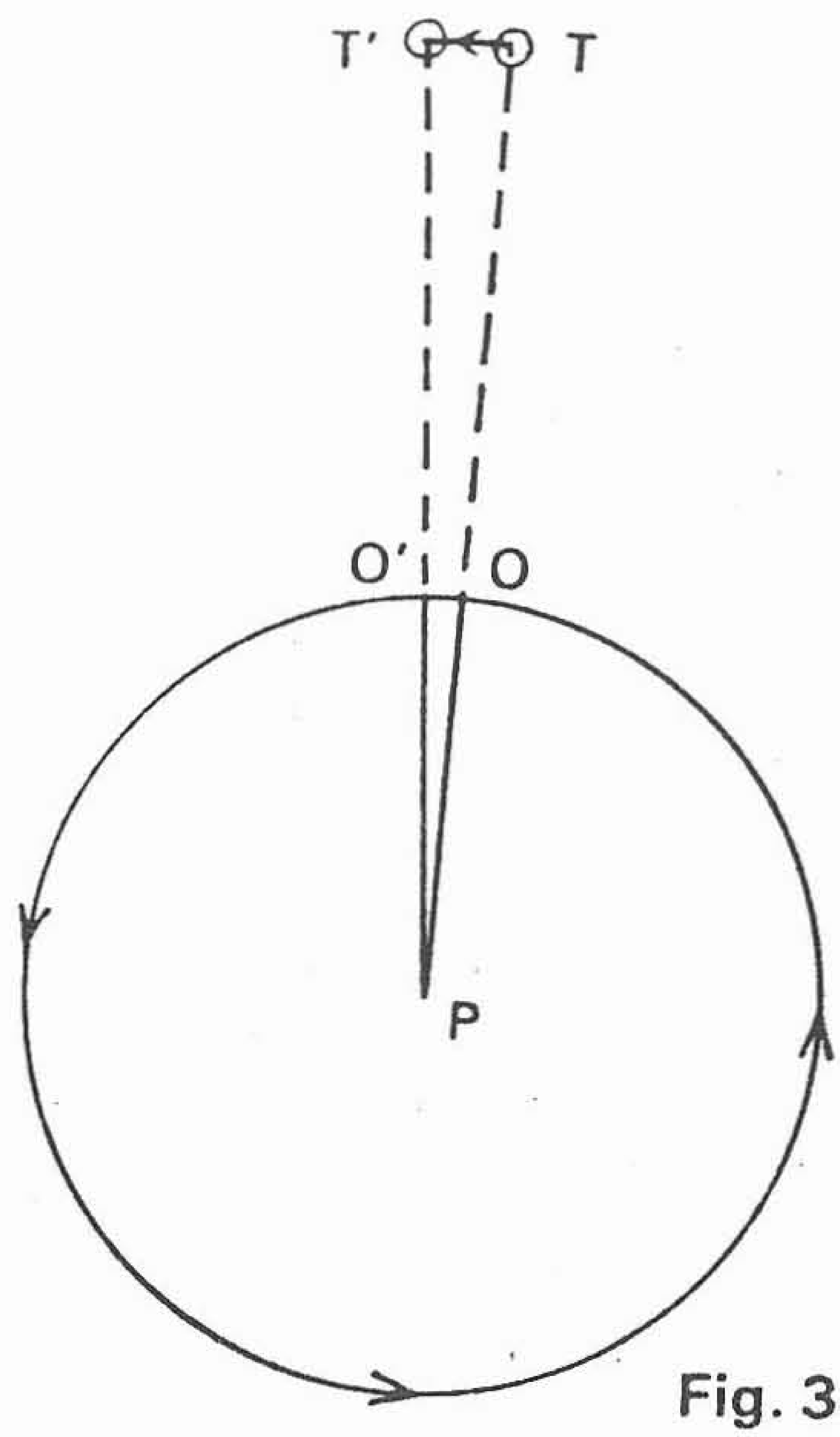


Fig. 38

Fig. 38 is drawn on the plane of the equinoctial.

P	reps the N Pole of the earth
O & O′	reps the positions of the observer.
PO & PO′	reps observer's meridian
T & T′	reps positions of true sun.

Imagine the True Sun (T) to be on the observer's meridian PO on a certain day. The earth rotates on the axis as indicated by the arrows on the circumference of the circle. After one complete rotation of 360°, when the observer is brought back to O, the Sun is no longer at T, but has moved out to T′. This is the apparent movement of the Sun due to the displacement of the earth on its own orbit, during that interval. So in order to bring the Sun on the meridian again, the observer has to rotate through a small arc OO′. Now that the True Sun is on the meridian again, one Apparent Solar day is complete.

The angular displacement TT′ or OO′ is not constant each day, because of Kepler's second Law of motion described in chapter III.

This law states, that radius vectors of planets in orbital motion, sweep out equal areas in equal time.

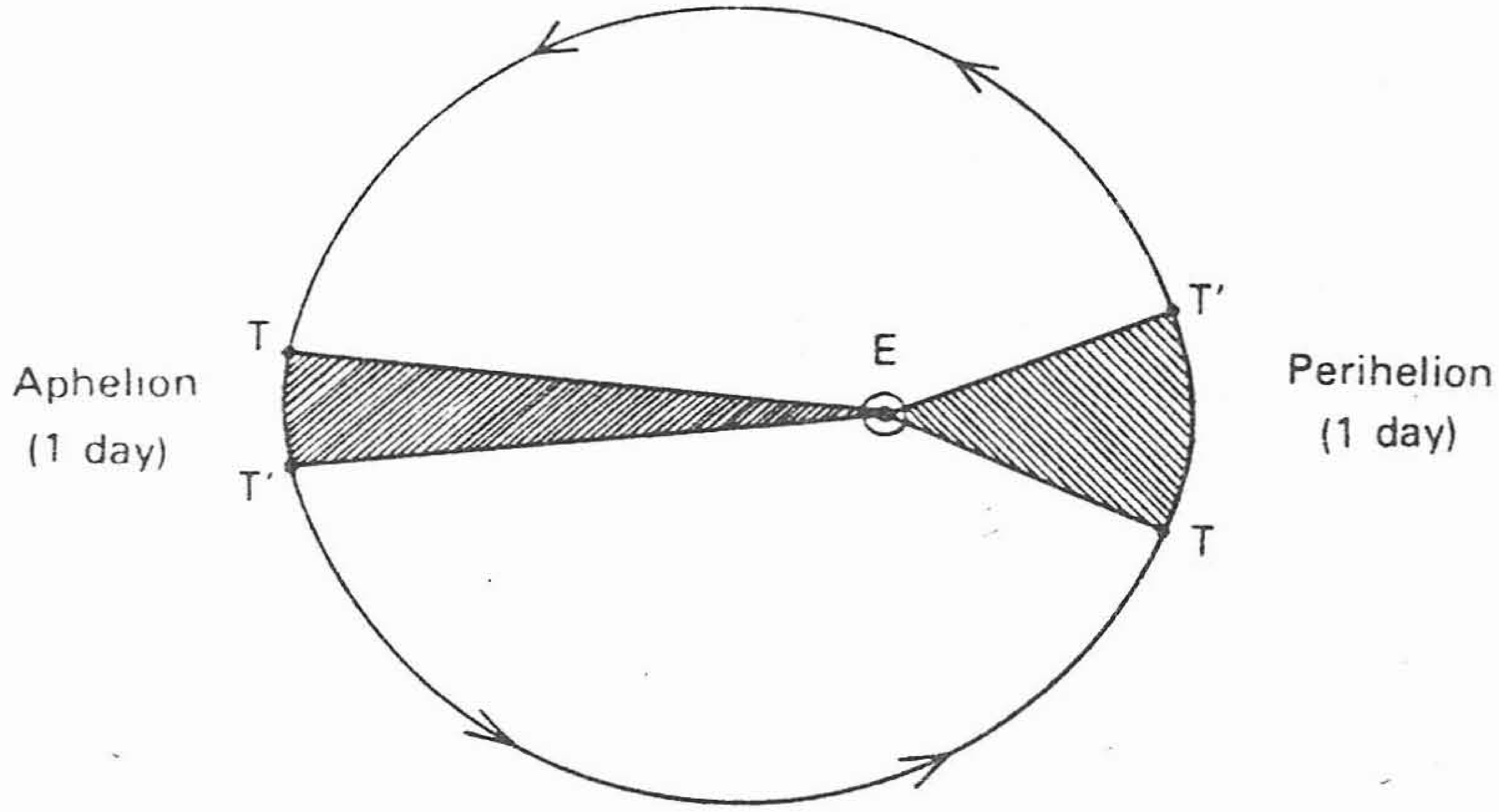


Fig. 39

For purposes of illustration, we can assume without any error that the Earth is at one of the focii of the elliptical orbit and the sun moving round the orbit. It will be seen from fig. 39 that the movement of the Sun T to T′ in one day at aphelion is a smaller arc than the movement TT' when in perihelion so as to keep the areas swept out by the radius vectors same at all times. TT' will be smallest at aphelion and largest at perihelion. At other times, it will be some where between these two values. From this it is evident that the length of an apparent Solar day will not be constant. It will have a minimum value at aphelion and maximum value at perihelion. Hence the true sun is not the best time keeper, as the length of the apparent solar day varies day to day.

To get over this difficulty, we introduce another fictitious Sun called the **"Mean Sun"** which moves at a constant rate equal to the average rate of motion of the true sun. Whereas the true sun moves at a varying rate along the ecliptic, the Mean Sun moves at a constant rate along the equinoctial.

Mean Solar day : is defined as the interval between two successive transits of the Mean Sun over the same meridian.

This has a constant length of 24 hours.

Refering to Fig. 40

Imagine the Mean Sun (M) to be on the observer's meridian PO, on a certain day. The earth rotates on the axis as indicated by the arrows on the circle and after one complete rotation of 360° when the observer is brought back to O, the Mean Sun has moved out to M', thus necessitating, the observer to be rotated further by the arc OO' to bring the Mean Sun (M') on the meridian again so as to complete a mean solar day. This interval between two transits is divided into 24 hours of Mean Solar time. This arc MM' or OO' has a constant value as shown below.

Total time taken for Sun to go round the earth once through 360° on the orbit = 365¼ day.

$$\text{In one day it will cover} = \frac{360}{365\frac{1}{4}} \text{ degrees}$$

Which is just under 1° or $3^m\,56^s$ of Mean Solar time.

Since the movement MM' is constant, day after day, the length of a Mean Solar day will be a constant quantity of 24 hours. The clocks we use show Mean Solar time, which is universally used for all civil purposes.

For purposes of distinction, however, the time reckoned by the True sun, we call Apparent time, and time reckoned by Mean Sun, we call Mean time.

In nautical parlance the time is expressed in 24 hours notation, OO hours being midnight and 1200 hours being noon and 2400 hours being next midnight. For civil purposes it is convenient to change date at midnight and commence a new date at OO hours.

Time measurment essentially, is an expression of the position of the Sun relative to the observer.

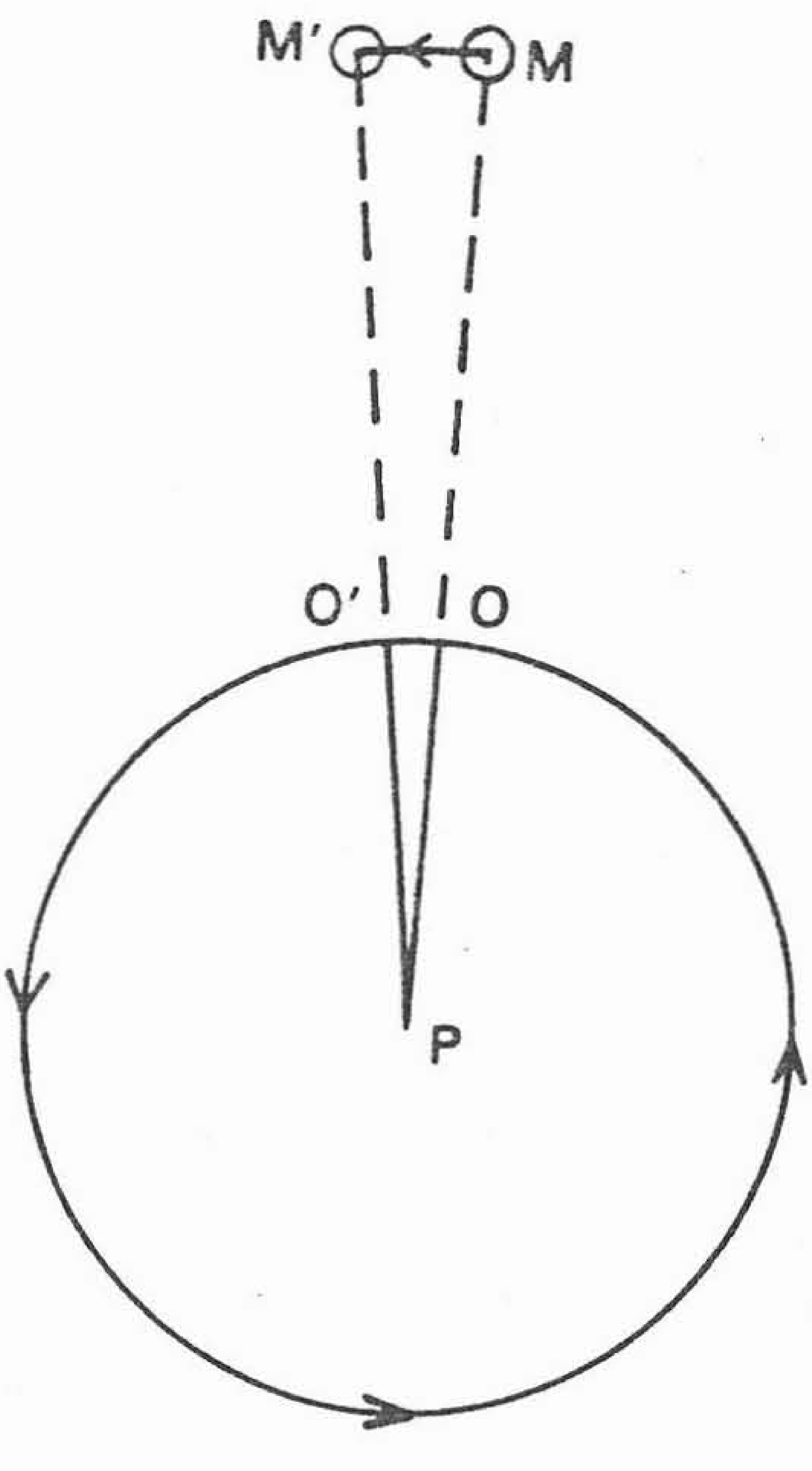


Fig. 40

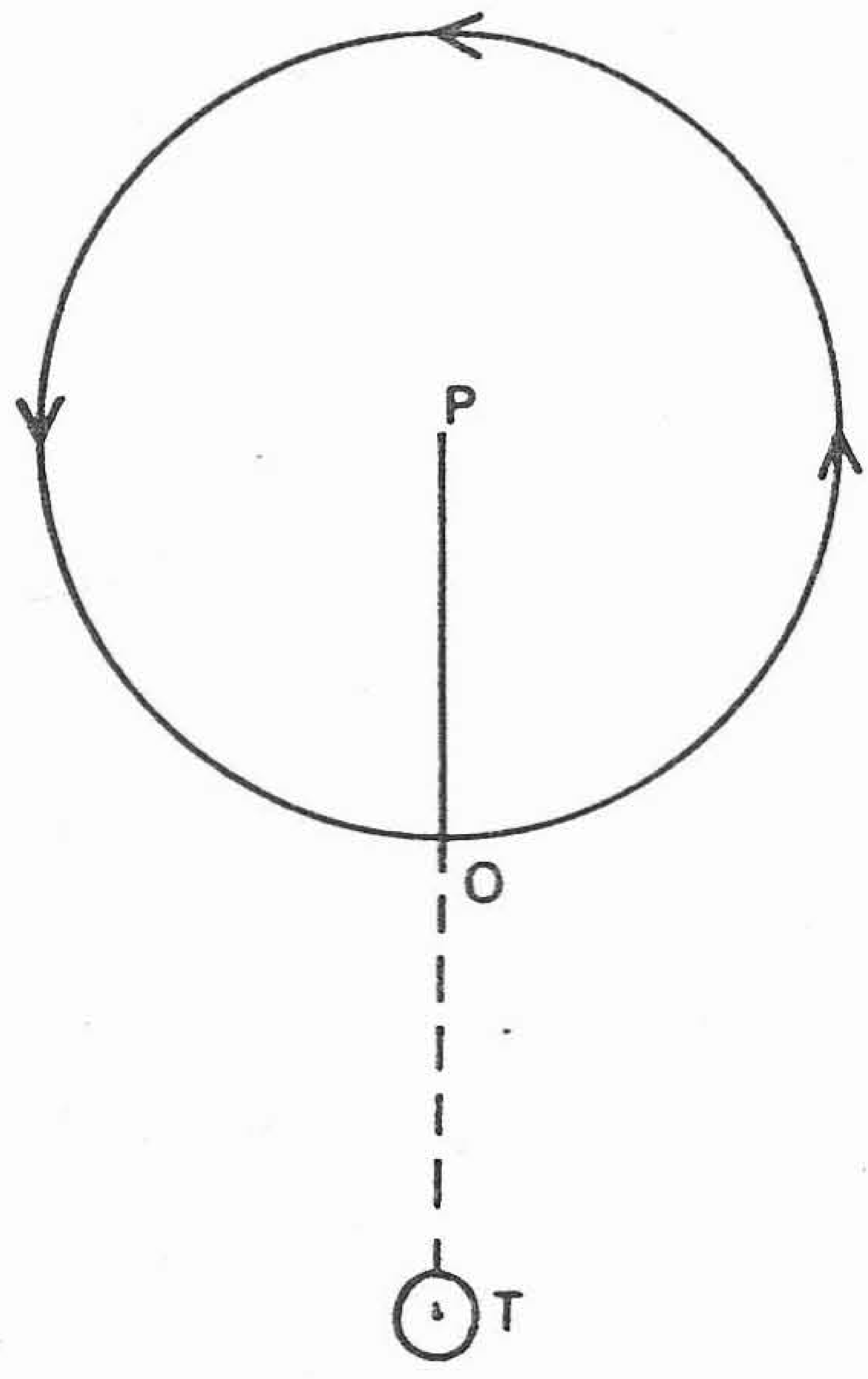


Fig. 41

Referring to fig 41, when the true sun (T) is on the observer's meridian PO, the H.A. of the sun is zero, but the time is noon or 1200 hours.

As the earth rotates on its axis and carries the observer away as shown by the arrows in fig. 41, the sun appears to move westwards. Say four hours later the situation will be as in figure 42.

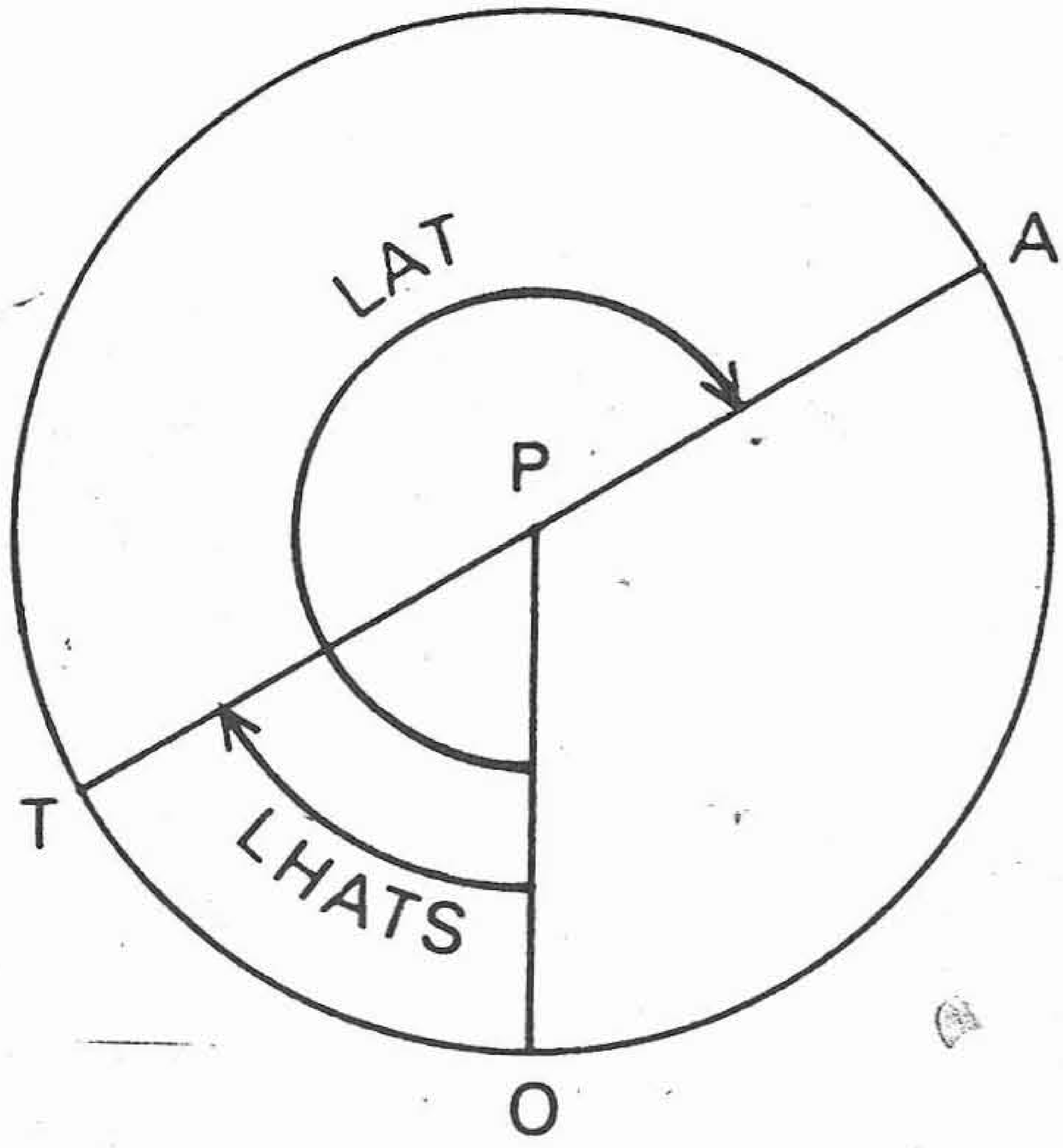


Fig. 42

O$\hat{P}$T reps local Hour Angle of the True Sun (LHATS) which this case is 4 hrs.

The local apparent time (LAT) now is 1600 hours. This is represented by the arc OTA or larger angle O $\hat{P}$ A.

Thus LAT = LHATS ± 12 hours.

Local Apparent Time (LAT) : is defined as the local hour angle of the point opposite to the True Sun. The difference of 12 hrs. occurs between LHATS & LAT (arc OTA in fig. 42 & 43) only because, we start the day off when the sun is on the antimeridian ie at midnight.

If we substitute Mean Sun (M) instead of the true sun, we can show, exactly the same way as aforesaid that,

LMT = LHAMS ± 12 hours.

Local Mean Time : (LMT) is defined as the local hour angle of the point opposite to the Mean sun. (Arc OMS in fig. 43).

The connecting quantity between the True sun & the Mean sun is the *Equation of time* (see fig. 43).

Equation of time : is defined as the small angle at the pole or the arc of the equinocital contained between the meridians of the true sun and the mean sun and expressed in time. It is a quantity to be applied to the position of the True Sun or it opposite point to arrive at the position of the Mean Sun or its opposite point respectively. The Equation of time can be positive or negative. The sign of Equation of time is given in the sense stated above. ie. if the mean sun is ahead (west) of the true sun eq. of time is positive; & if the mean sun is behind (east) of the true sun the eq. of time is negative.

In short this means

LMT — LAT = ±Equation of time.

On the daily page of the nautical almanac the quantity of equation of time is tabulated for 00 hours & 12.00 hours GMT on each day along with the L.M.T. of meridian passage of the Sun. It will be noticed therein, that it does not carry a sign with it, but the sign is implied.

Since LMT meridian passage is given for each day & LAT meridian passage is always 1200 hours :-

LMT Mer Pass - 1200 = Eq of time.

if LMT > LAT Eq. of time is + ve

if LMT < LAT Eq. of time is - ve

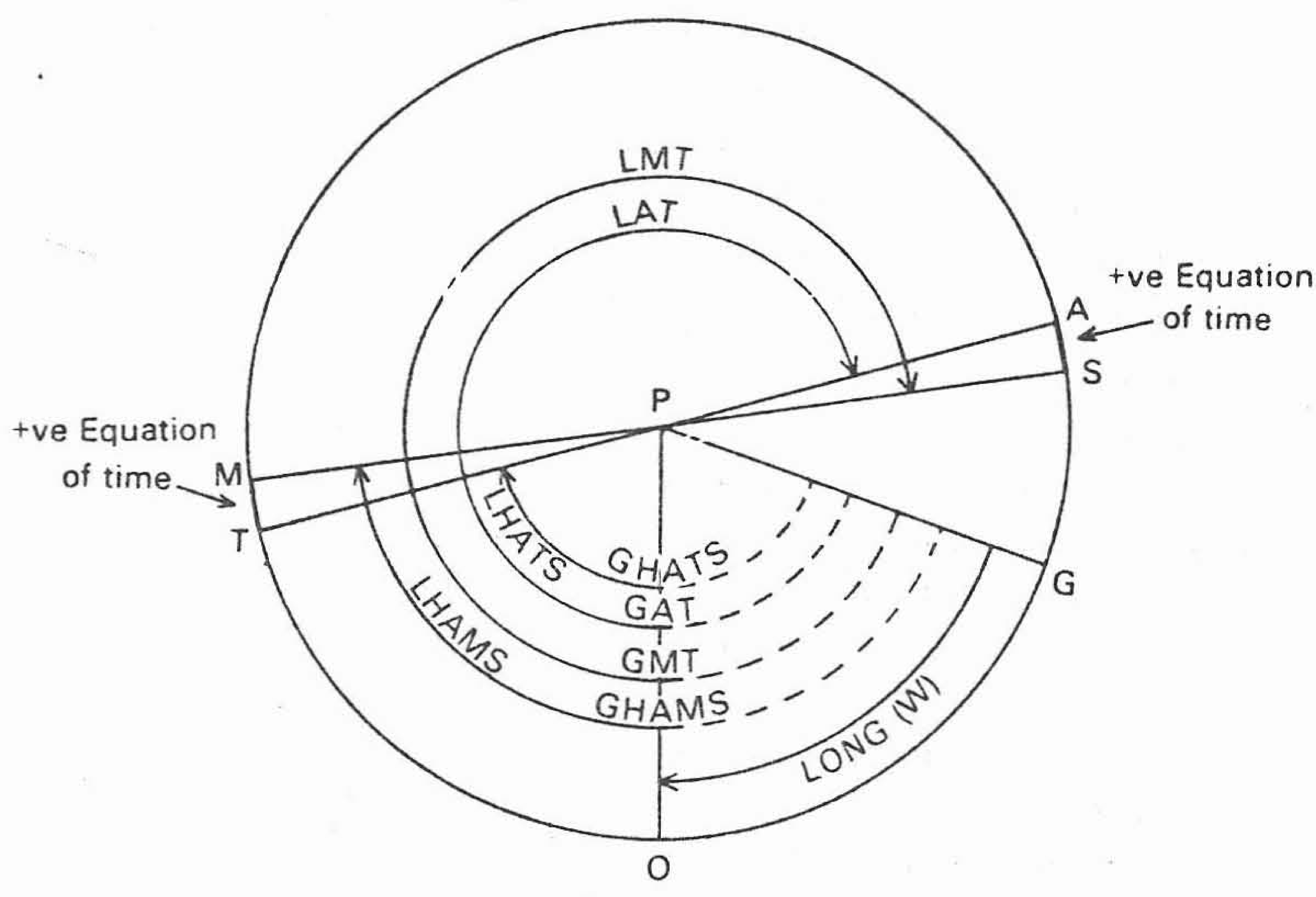


Fig. 43

Referring to fig. 43, if we measure all these angles, from the Greenwich meridian (PG) instead of the Local meridian (PO) we see that

GAT = GHATS ± 12 Hours

GMT = GHAMS ± 12 Hours

The difference between GAT & LAT or GMT & LMT is always the arc (GO) which is the Longitude of the observer.

GHA $>$ LHA Long West.

GHA $<$ LHA Long East.

There is a little rhyme which helps to remember this

GHA Best Long West

GHA LEast Long East

Local time : is the time corresponding to the longitude of the observer. As he changes longitude, his local time will change. This necessitates adjusting the clocks each day at sea, advancing the clocks as he proceeds eastwards and retarding the clocks as he changes the longitude westwards. It is customary at sea on merchant ships to set the clocks to show correct L.M.T. at noon of each day. This time we call the **ship's time.**

Standard time : is a time which is kept throughout a specified country, based on the LMT of a specified standard meridian suitable to that country. This becomes necessary in order to have a uniform time throughout that country or in certain areas of the country. Countries which have a large east west stretch such as, U.S.S.R., U.S.A., Canada, Australia etc. have different such standard time for different areas. The time difference between their standard time & GMT are tabulated in the nautical almanac for all countries.

Zone time : since the length of a mean Solar day is 24 hours during which time all the 360° of longitude on the earth are rotated on the axis once, the hour angle of the sun increases at the rate of 360/24 = 15° per hour. Hence every 15° of longitude, the time will differ by one hour. For uniformity of time keeping, particularly in the Navy the system of keeping zone time is followed. For this purpose the earth is divided into 24 zones of 15° longitude apart. Zero zone being 7½° of longitude on each side of the Greenwich meridian and each 15° of long thereafter on either side will be ± one hour zone etc. as shown in fig 44

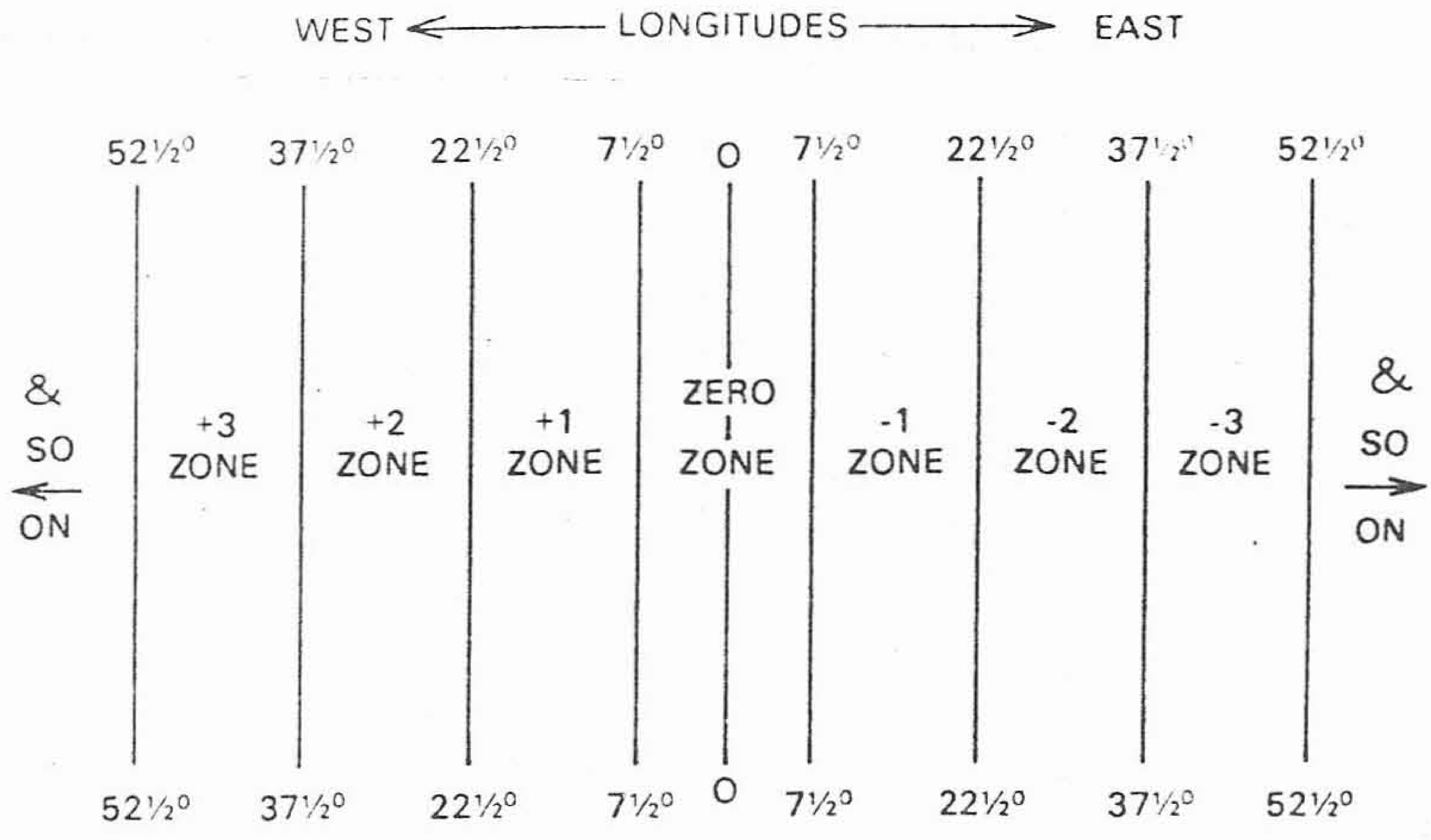


Fig. 44

All ships within a specific zone keep the same time as indicated by the time zone. Ships in zero zone keep G.M.T.

It will be observed that all Easterly zones are negative & Westerly zones are positive. The sign is indivative of the number of hours to be added or subtracted from the zone time to arrive at GMT.

If we keep dividing the earth likewise when we reach 180° longitude again there will be a Zone of 7½° longitude on each side of 180° as shown in fig. 45. This is the 24th zone, consisting of 7½° of - 12 zone & 7½° of + 12 zone.

Change of Date when crossing the 180° meridian

Assume the time at Greenwich is 0300 hours on 5th of a month. Ships in - 12 zone will be showing a time 1500 hours on the 5th whereas ships in + 12 zone will show a time of 1500 hours on 4th (see fig. 45). Hence we see that if a ship crosses the 180th meridian on an Easterly course she has to retard one day and if she crosses 180° meridian on a westerly course she has to advance one day in order to match with the calender date.

If a ship starts from Greenwich meridian and circumnavigates the earth on an easterly course advancing time each day, when she reaches Greenwich meridian again, she would have

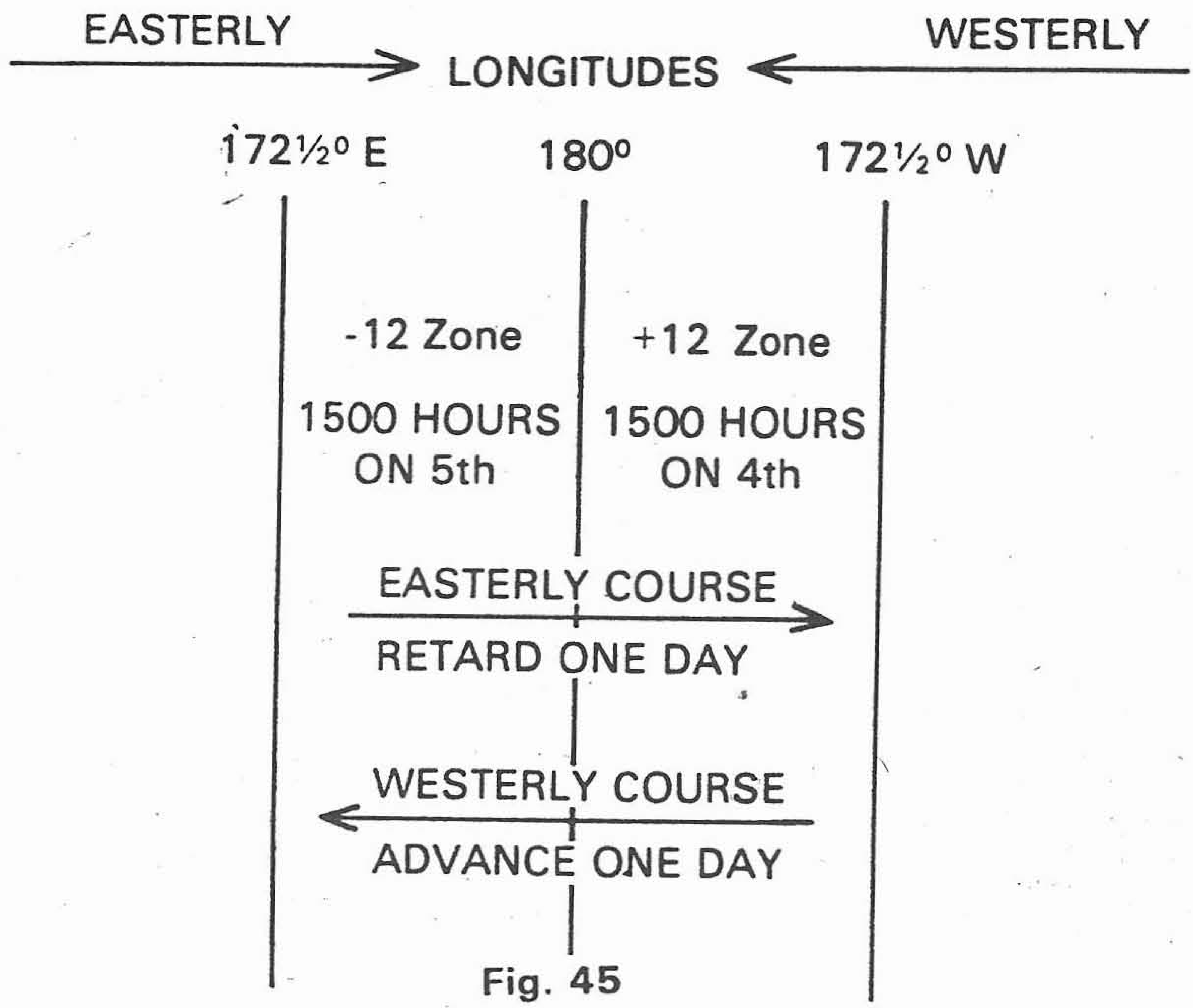


Fig. 45

advanced 24 hours of time and her calendar date would have been one day ahead of Greenwich date. Conversely if she circumnavigated on a westerly course she would have retarded her clocks by 24 hours. When she reaches Greenwich meridian again, her calendar would be one day behind Greenwich date. It is to avoid such an anomoly that we have to change the date as the ship crosses the 180° meridian.

International date line :

The 180° meridian passes through a few islands in the ocean and, it is inconvenient for civil purposes to have two dates on the same island. Hence the 180° meridian in suitably modified so as to allow the islands to fall wholly, either East or West of the modified line. This modified 180° meridian is called the *International date line.* For major part of its length the International date line coincides with the 180° meridian. The actual change of date takes place in fact when crossing the International date line. This line is marked on the chart of th area, and the positions through which it passes are also indicated in the nautical almanac after the standard time tables.

Equation of time becomes zero four time a year.

We saw earlier in the chapter that the Equation of time is sometimes positive and sometimes negative, depending on whether the mean sun is west of the true sun or east of the true sun In this process of changing from positive to negative or vice versa, it becomes zero. This occurs four times a year. The reasons why Eq. of time becomes zero four times a year is explained below.

We know that the true sun moves at a varying rate along the orbit which is in the same plane as the ecliptic whereas the mean sun moves at a constant rate along the equinotial. By defination Eq. of time is the angle at the pole between the meridians of the mean & true Sun. When Eq. of time becomes zero, both these suns are on the same meridian and hence does not produce any angle at the pole. Since the track and the rate of motion of both the Suns are different, it is not directly possible to compare the movement of Mean & True suns. It is therefore necessary to introduce another fictitious sun called the **"Dynamical Mean Sun"**. This imaginary sun is assumed to move at a constant rate (ie. at the average rate of the true sun) along the ecliptic. Thus the rate of motion of the Mean sun & Dynamical mean sun are the same, but moving on different great circles. All the three suns take excatly one year to go round the earth.

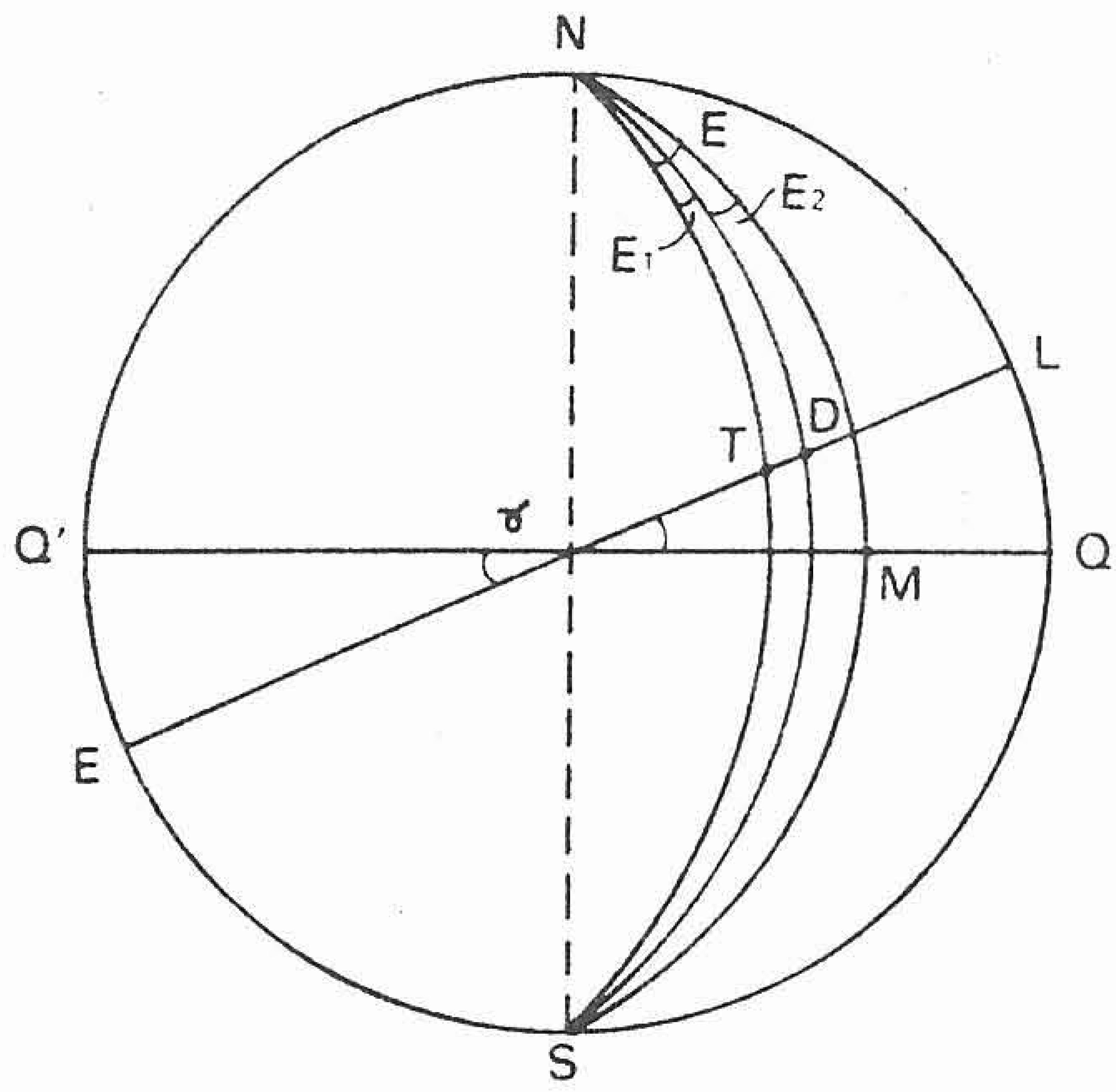


Fig. 46

NS - Meridian of ♈
Q'Q - Equinocitial
EL - Ecliptic.
D - Dynamical Mean sun.
M - Mean Sun.
T - True sun.
$L\hat{♈}Q = Q'\hat{♈}E$ -(obliquity of the ecliptic.)

Let us say for convenience we start all of them together at first point of Aries. Since all suns are on the same meridian Eq. of time is zero to start of with. (refer to fig. 46)

After a certain number of days, Mean Sun would have reached point 'M' and the Dynamical Mean Sun would have reached point 'D'. Since their rate of motion is the same arc ♈M =♈D but their meridians are not same, because the meridians are drawn from the pole of the earth. The small angle that is produced at the poles, between the two meridians, is one component of Eq. of time marked E_2 in figure. The quantity E_2 is called the **component due to the "Obliquity of the ecliptic."** When Mean Sun reaches point 'Q' the Dynamical mean sun will reach point 'L' since ♈Q = ♈L. Point 'L' is the Summer Solstic position of the sun. Here again the meridians of D & M are same, and the component E_2 is zero. Similarly 90° further away ie. when D & M reaches first point of Libra (♎) their meridians will again coincide. Same thing will repeat when 'D' reaches point 'E', & 'M' reaches Q'. Thus we see the component due to the obliquity of the ecliptic vanishes four times ie. at Equinoxes & Solstices.

Let us now compare the movements of the True Sun and the Dynamical mean sun. We know that the True Sun (T) moves at a varying rate along the elliptical orbit; fastest at perihelion and slowest at aphelion. We also know that the Dynamical mean sun moves at a constant rate along the ecliptic, which is a great circle in the same plane as the orbit.

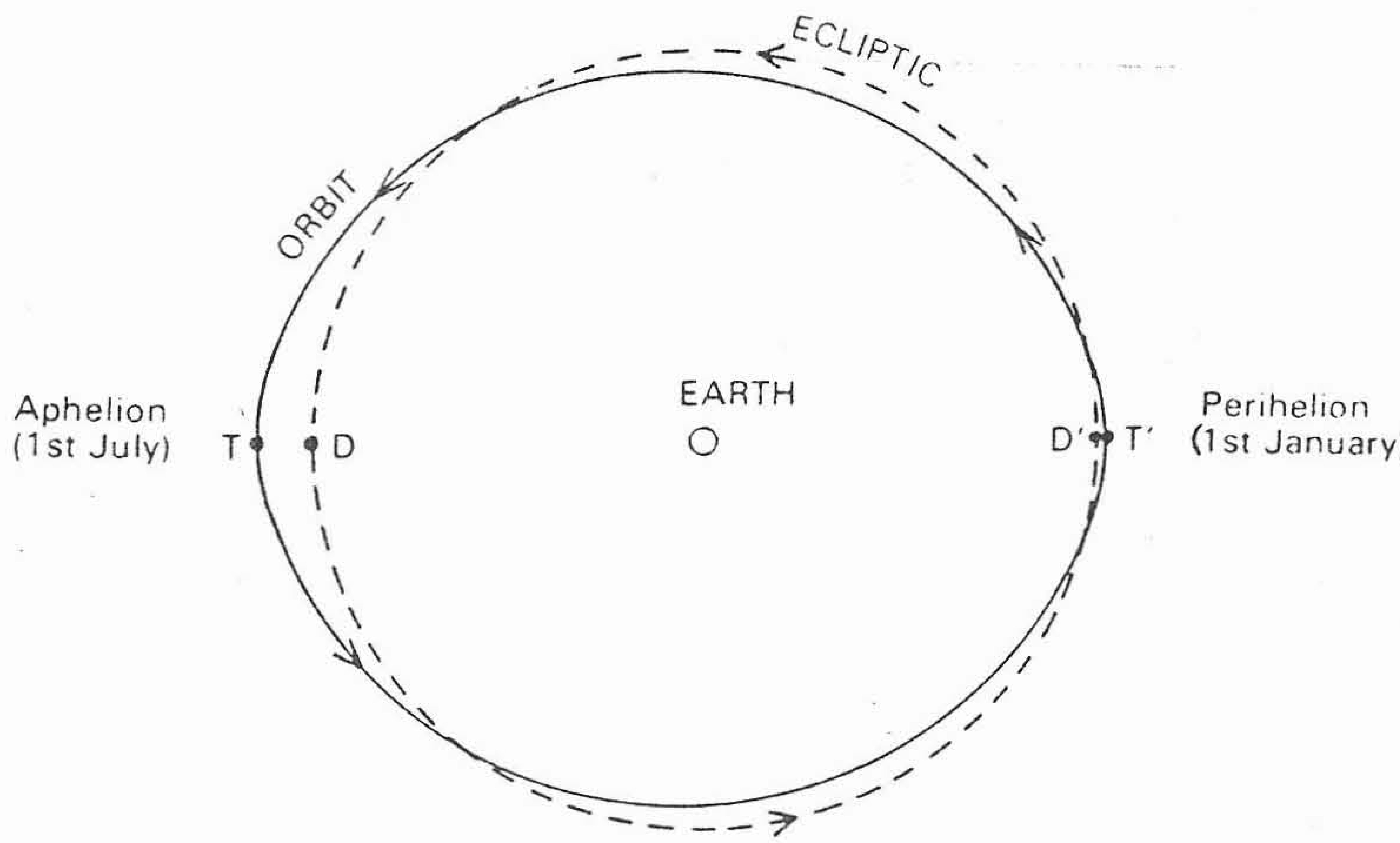


Fig. 47

In fig. 47

D & D′ reps Positions of Dynamical Mean Sun at aphelion a perihelion respectively.

T & T′ reps positions of True sun at aphelion & perihelion respectively.

The direction of movement of the Suns is indicated by arrows.

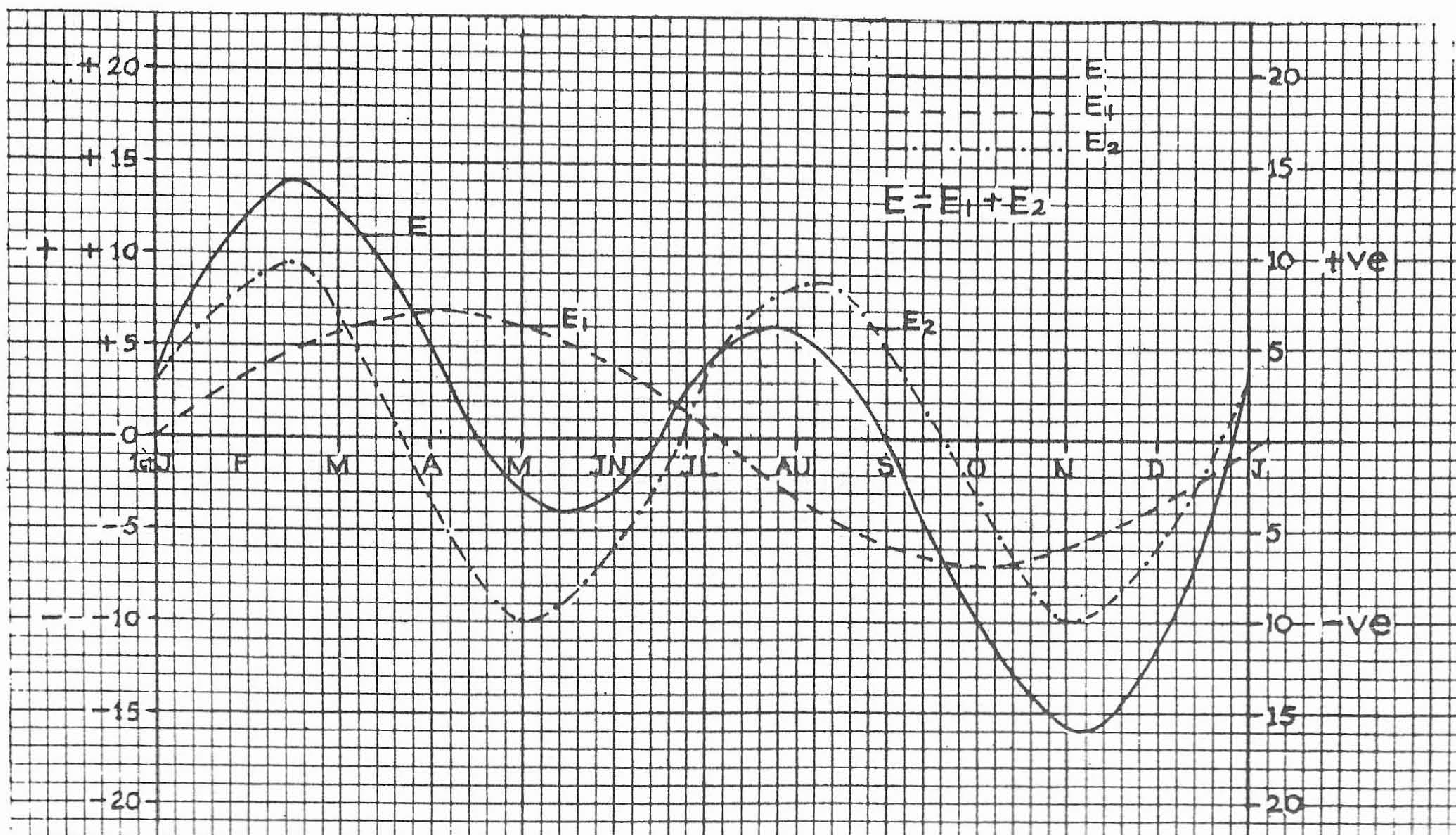
At aphelion, the true sun is moving at its least speed. The Dynamical mean sun moving at constant speed, overtakes the True Sun at this point. Both the suns will then be on the same meridian. (Points D & T). Similarly at Perihelion, the True sun moving at its fastest rate overtakes the Dynamical mean sun at that point (Points D′ & T′). Again both are on the same meridian. From perihelion to aphelion, though the true sun initially gains on the Dynamic mean sun, but loses speed as it approaches aphelion, letting the Dynamical mean sun overtake it at aphelion. From aphelion to perihelion, though the Dynamical mean sun initially gains on the true sun, but loses as it approaches the perihelion thus letting the true sun overtake, the Dynamical mean sun at perihelion. Thus except at aphelion and perihelion both the suns are on different meridians, causing a small angle at the pole between their meridians. This quantity is the other component of equation of time called the **component due to Eccentricity of the orbit,** marked E_1 figure 47. This component becomes zero at Aphelion and Perihelion.

The equation of the time (E) tabulated in the nautical almanac is the algebrical sum of E_1 & E_2

$$E = E_1 + E_2$$

If the values of E_1 & E_2 for the whole year are calculated and plotted on a graph it can be shown that their combined value, the equation of time (E) becomes zero four times a year, not at equinoxes and solstices, nor at aphelion and perihelion but at four other times viz. mid-April, mid-June, first of September and 25th December. This is illustrated in the following graph (fig. 48). It will also be evident from the graph, that equation of time has two positive maximums during mid-Fibruary & mid-July and two negative maximum during mid-May & early November each year. The exact date & time at which it become zero for a specific year has to be taken from the nautical almanac for that year.

(See graph on page 81)



+ 20
+ 15
+ 10
+ 5
0
-5
-10
-15
-20
+
−
1stJ
F
M
A
M
JN
JL
AU
S
O
N
D
J
20
15
10
5
5
10
15
20
+ve
−ve
E
E₁
E₂
E
E₁
E₂
E = E₁ + E₂

Fig. 48

Excercise VII

(1) Define the following :-
Apparent Solar day, Mean Solar day. LAT., LMT, Equation of time Dynamical Mean Sun, zone time.

(2) Why is it that, the true sun cannot be used as an accurate and constant time keeper ? Explain.

(3) Show with aid of suitable diagrams the truth of the following statements.
GMT - W. Long = LMT
LHATS $\pm$ Eq. of time = LHAMS
LMT - LAT = $\pm$ Eq. of time.
GHATS $\pm$ 12 hours = GAT

(4) No sign is indicated for the tabulated value of equation of time in the nautical almanac. Why ? How is the sign determined ?

(5) Why is it necessary to change the date when crossing 180° meridian and how is the change effected ?

(6) What is the "International date line" ? Where & how would you find this ?

(7) Explain why equation of time becomes zero four times a year.

(8) What is the relationship between GHA of the Sun and corresponding GMT - Discuss.

CHAPTER VII

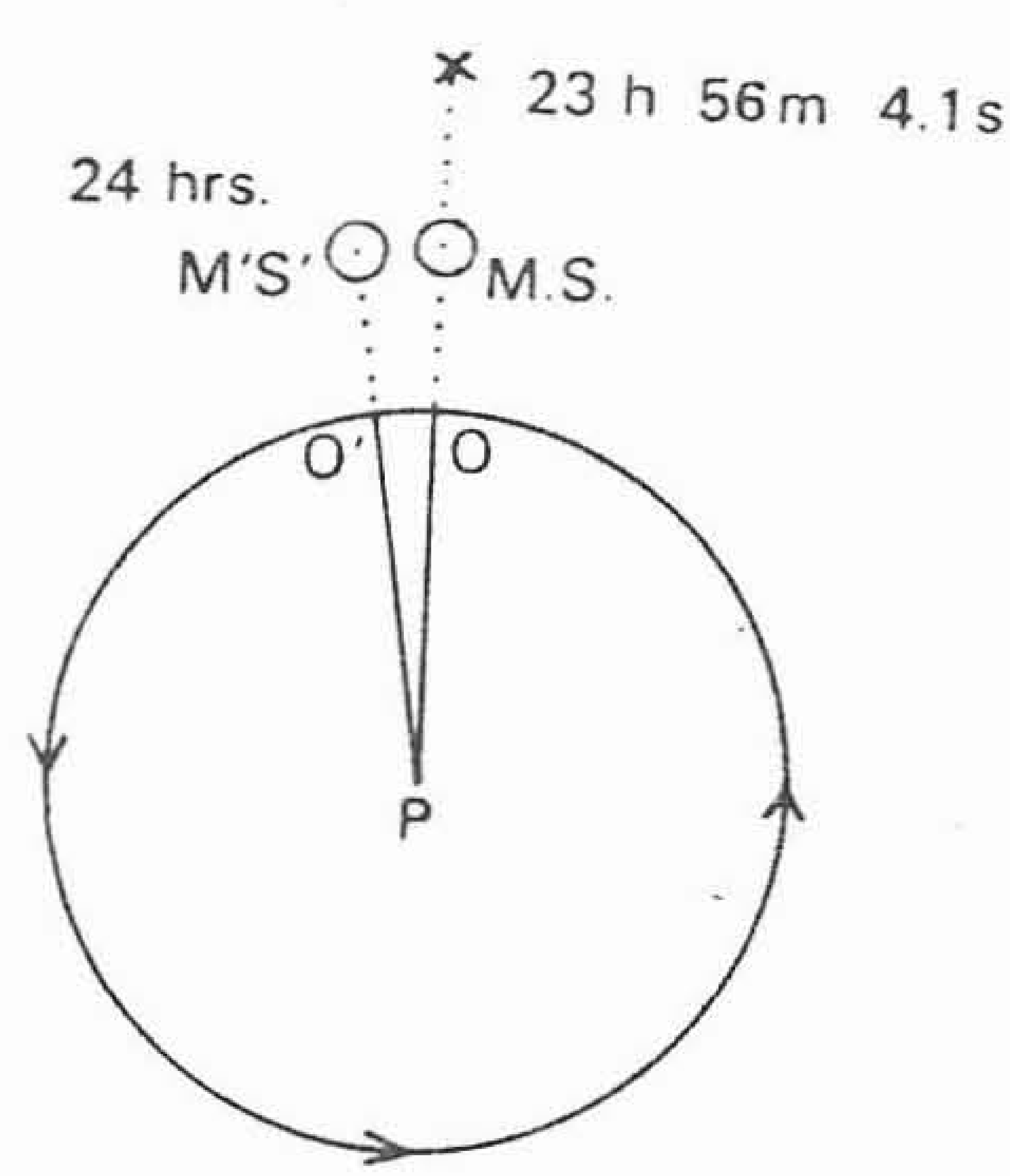
SIDEREAL TIME

Sidereal time is the time reckoned by stars. Unit used for measurement of Sidereal time is a Sidereal day.

Sidereal Day is defined as the interval between two successive transits of a star over the same meridian. This has a constant value. Since, this is too large a unit for practical purposes, it is sub-divided into 24 Sidereal hours, minutes and seconds, which is equivalent to 23h 56m 4.1s of mean solar time.

Even though any star could be used for reckoning Sidereal time, conventionally, the "First point of Aries" is used for this purpose. Though this point cannot be visually observed, being a fixed point, it not only behaves exactly as any star but also has the advantage of being above the horizon, for exactly half a Sidereal day, for all observers on the earth, because of the fact it is on the equinoctial and hence, has no declination.

For civil purposes, all clocks show mean solar time. Because of this, it would appear that all star rise culminate and set four minutes earlier each day; since we are in fact comparing its diurnal motion in relation to the sun.



MS	-Mean Sun
X	- A Star
PO & P O'	- Observer's meridian
P	- N. Pole of the earth
O	- Observer.

On a certain day : Let the Mean Sun (MS) and a Star (X) in fig. 49 be on the observer's meridian, PO.at the same time. The observer is rotated on the earth's axis as shown by the arrows. After an interval of 23 h 56 m 4s of Mean solar time, the observer is brought back to O' i.e. with the star again on the meridian, thus completing a Sidereal day. In that interval of time, the mean sun MS has moved to M'S' in keeping with its daily orbital motion, which necessitates the observer (O) to rotate through a further arc of roughly one degree to bring the sun on the meridian to complete the mean solar day of 24 hours. Thus a Mean solar day is longer than a Sidereal day by 3m 56 seconds of mean solar time. Hence a star crosses the meridian nearly 4 minutes earlier each day. Consequently, it rise and sets also four minutes earlier each day. If a star behaves this way, so will the First point of Aries behave. However the times of rising & setting of stars are further governed by the latitude of the observer and the Declination of the star.

Hence, 24 hours of Sidereal time = 23 h 56 m 4.ls of mean solar time.
ie. 86400 seconds of Sidereal time = 86264.1 seconds of Mean solar time.

This relationship gives us a yardstick to convert Mean solar time into corresponding Sidereal time or vic-versa.

To convert Mean Solar time into corresponding Sidereal time we multiply M.S.T. by

$$\frac{86400}{86264.1} = 1.00274$$

Sidereal time	=	Solar time x 1.00274
Solar time	=	Sidereal time x 0.99727

Example :

1 Convert 14 h 32m 18s of Sidereal time into Mean solar time.

14h 32m 18s of Sidereal time = 52338 sidereal secs.

Solar time = 52338s x 0.99727 = 52195.1 Mean Solar secs.

= 14h 29m 55.1s M. S. T.

2. Convert 8h 52m 36s of Mean Solar time into corresponding Sidereal time.

8h 52m 36s of M.S.Time = 31956 sec of Mean Solar Time

Sid. time = 31956 x 1.00274 = 32043.6 seconds of Sid. time

= 8h 54m 3.6s of Sid.time.

It is evident from the above that the length of a Sidereal hour, minute or second is slightly shorter than the corresponding Mean solar hour, minute or second.

Unlike a Solar day, which starts off when the Sun is on the midnight or antimeridian of the observer, a sidereal day starts when the First point of Aries (♈) is on the observer's meridian.

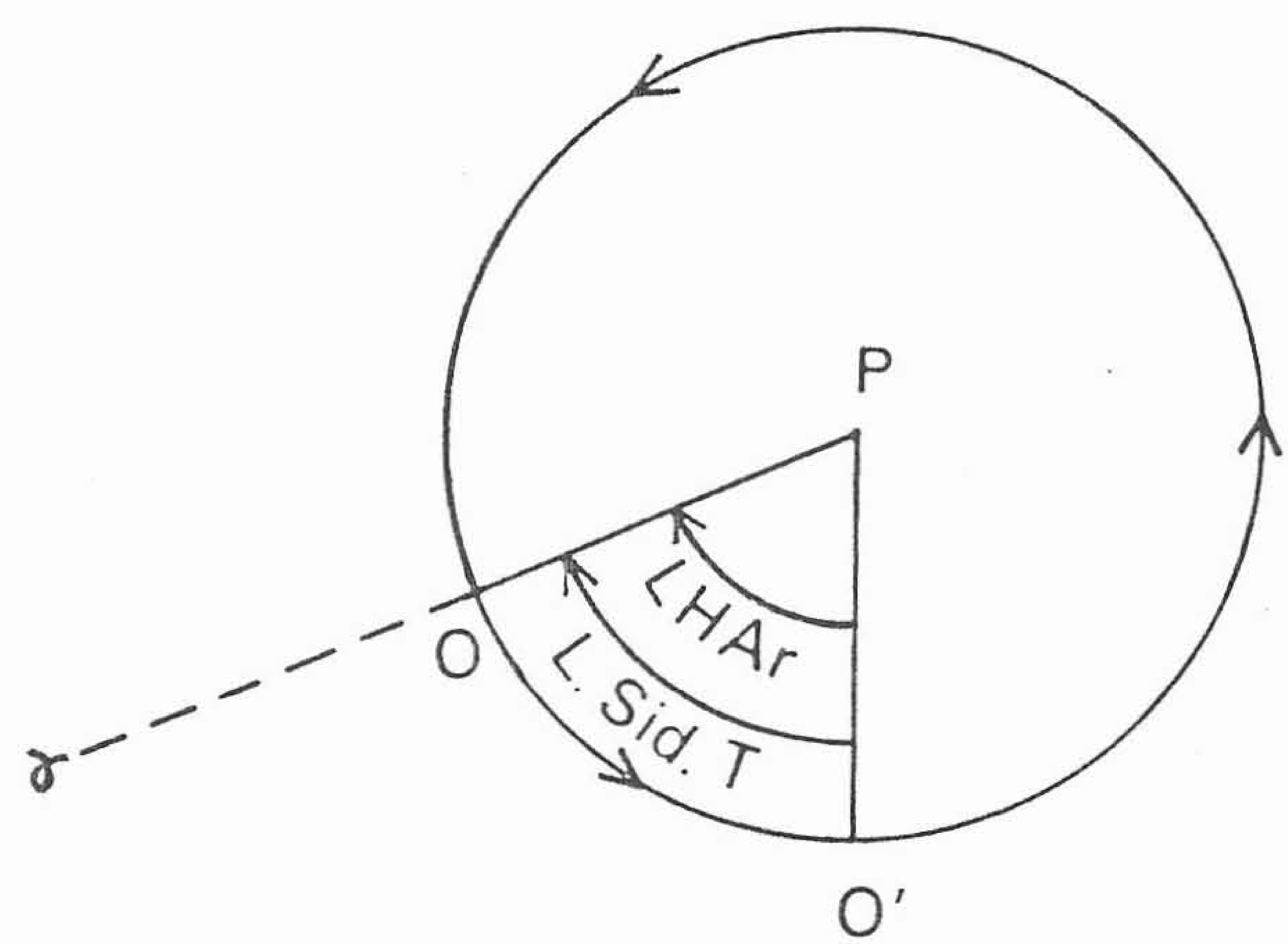


Fig. 50

On a certain day let '♈' be on the observer's meridian (PO). The Sidereal day commences then. The earth rotates on its axis as shown by the arrows in fig. 50. After a certain interval of time, say the observe is brought to a position (O') in fig. PO' being his meridian. Aries, being a fixed point, appears to the observer that it has increased its H.A. by arc O'O.

The arc O'O is the L H A '♈' at that time. This arc is also equal to the local Sidereal time. The only difference is LHA '♈' is expressed in arc whereas Sidereal time is expressed in time.

∴ LHA '♈' (in arc) = Local Sidereal time (in time)

This Sidereal time when converted to the corresponding mean solar time, will indicate, the number of hours, minutes and seconds which has elapsed after '♈' was on the meridian of the observer.

Since our clocks do not keep Sidereal time, for civil purposes, the conversion of Sidereal time to Solar time or vice versa, remains purely an academical and arithmetical exercise and has little practical value. Though we do use Sidereal time constantly for all stellar observations, we use it as L H A '♈' and not as Sidereal time in the conventional sense.

The following discussion illustrates its practical use.

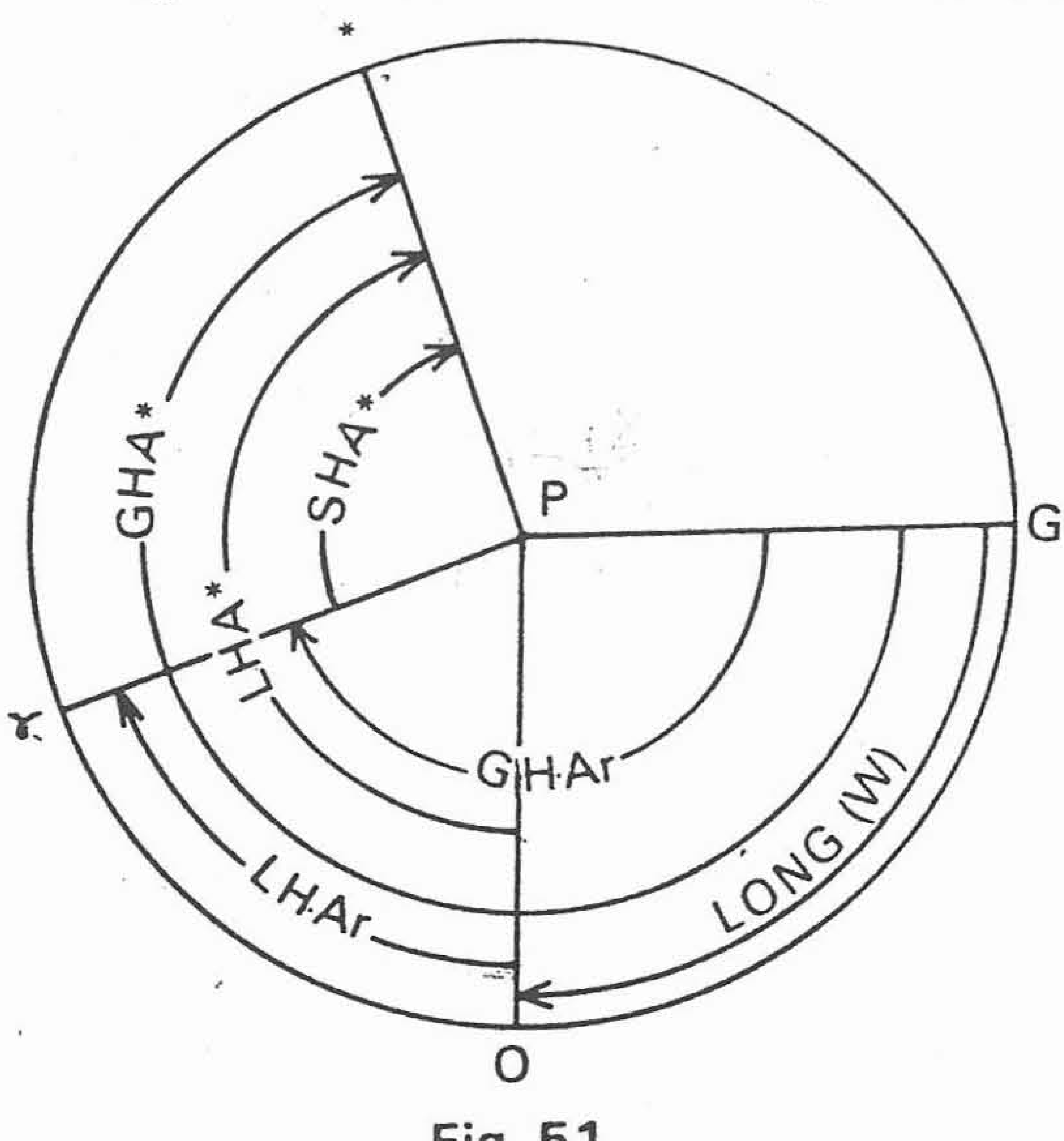


Fig. 51

in fig. * reps a star

PO	–	Observer's meridian
PX	–	Meridian of star
P♈	–	Meridian of ♈
PG	–	Meridian of Greenwich
Arc O♈	–	LHA ♈
Arc♈ *	–	SHA *
Arc O*	–	LHA *
Arc GO♈	–	GHA ♈
Arc GO*	–	GHA *
Arc GO	–	Long of observer (W)

Arc O♈ + arc ♈* = arc O♈ *'
ie. LHA ♈ + SHA * = LHA *
Arc GO ♈ + arc ♈ * = arc G ♈ *
ie. GHA ♈ + SHA * = GHA *

It is evident from this that GHA ♈ ~ LHA♈or GHA* ~ LHA* gives arc GO which is the longitudeof the observer.

GHA > LHA Long. is West.
GHA < LHA Long is East.

GHA ♈ is tabulated in the nautical almanac for each hour of GMT for each day. By applying the increment to GHA ♈ we can find the GHA ♈ for any GMT of observation. Knowing the longitude we can compute the LHA ♈ for that time viz :
GHA ♈ ± Long = LHA ♈ (– for West Long) (+ for East Long)
LHA ♈ + SHA * = LHA *
The LHA * is what is required for all stellar observations.

The nautical almanac also gives at the bottom of the column for GHA ♈ a parameter called "Meridian passage time of ♈" This is the LMT meridian passage of ♈ on the Greenwich meridian for the middle date on that page. Since we know that stars,also ♈"come on the meridian 4 minutes earlier each day LMT mer. pass. for the previous day would be 4 minutes later and for the following day will be 4 minutes earlier than the tabulated value. This time defines precisely the position of the mean sun, when ♈ is on the Greenwich meridian. Since, we have already seen how mean time is defined, we can say, that mer. pass. of ♈ is nothing but the hour Angle of the point opposite to the mean sun, when ♈ is on the Greenwich meridian. This is illustrated in fig. 52.

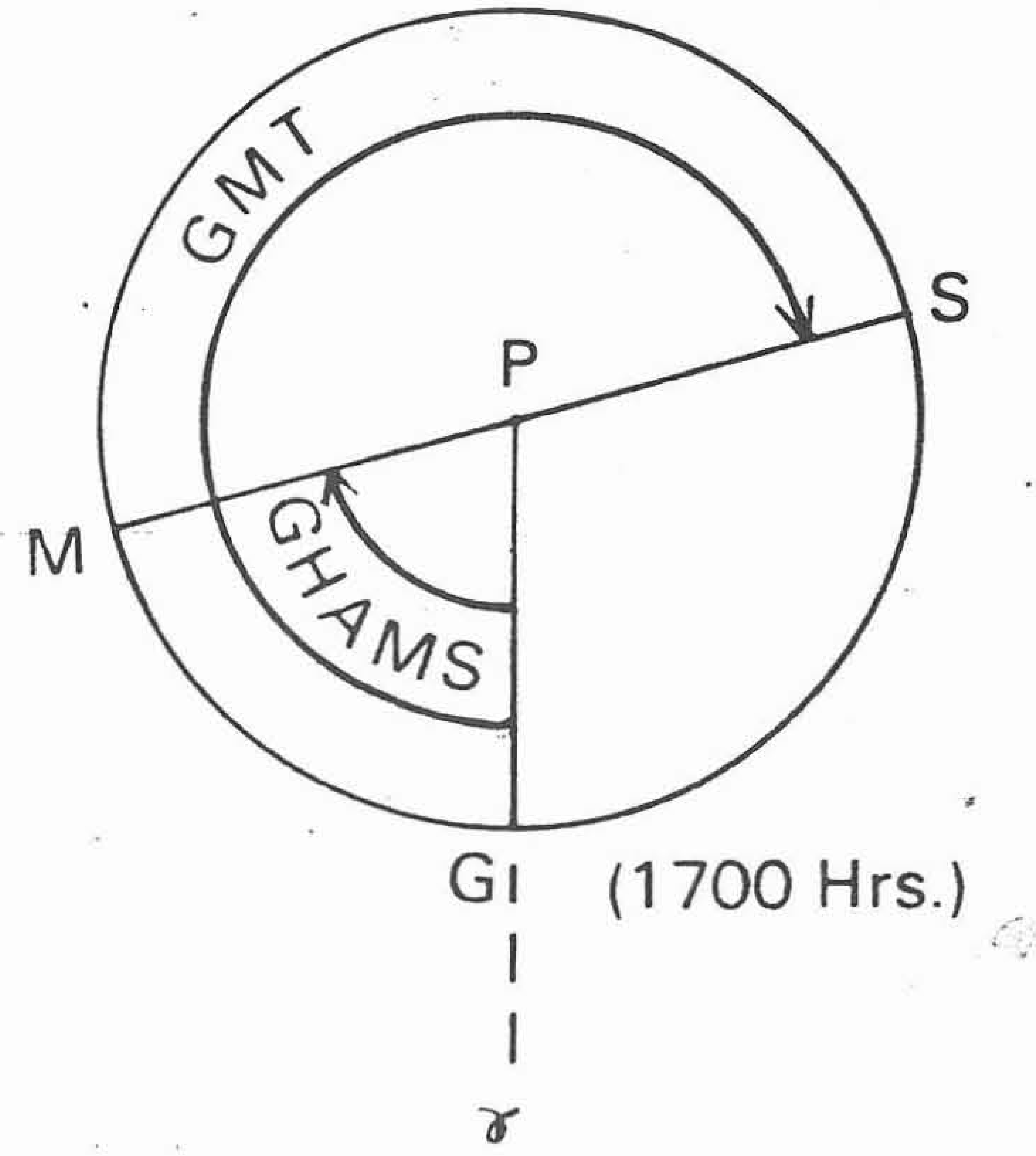


Fig. 52

Say on a certain day that LMT mer. pass. of ♈ at Gr.
was 1700 Hours
Figure shows ♈ on Gr. meridian
PM is the meridian of the mean Sun
Arc ♈ M = GHAMS
Arc ♈ MS = GMT

If the ship's longitude is substituted for the Greenwich meridian the same truth will still hold good viz :

LMT mer. pass. of ♈ at Ship is the LHA of the point opposite to the mean Sun, when ♈ is on the observer's meridian. But LMT meridian passage of ♈ for a given longitude is obtained from the LMT mer. pass. of ♈ at G after applying a small correction called the **Longitude Correction.** The principal of applying this correction is shown below :

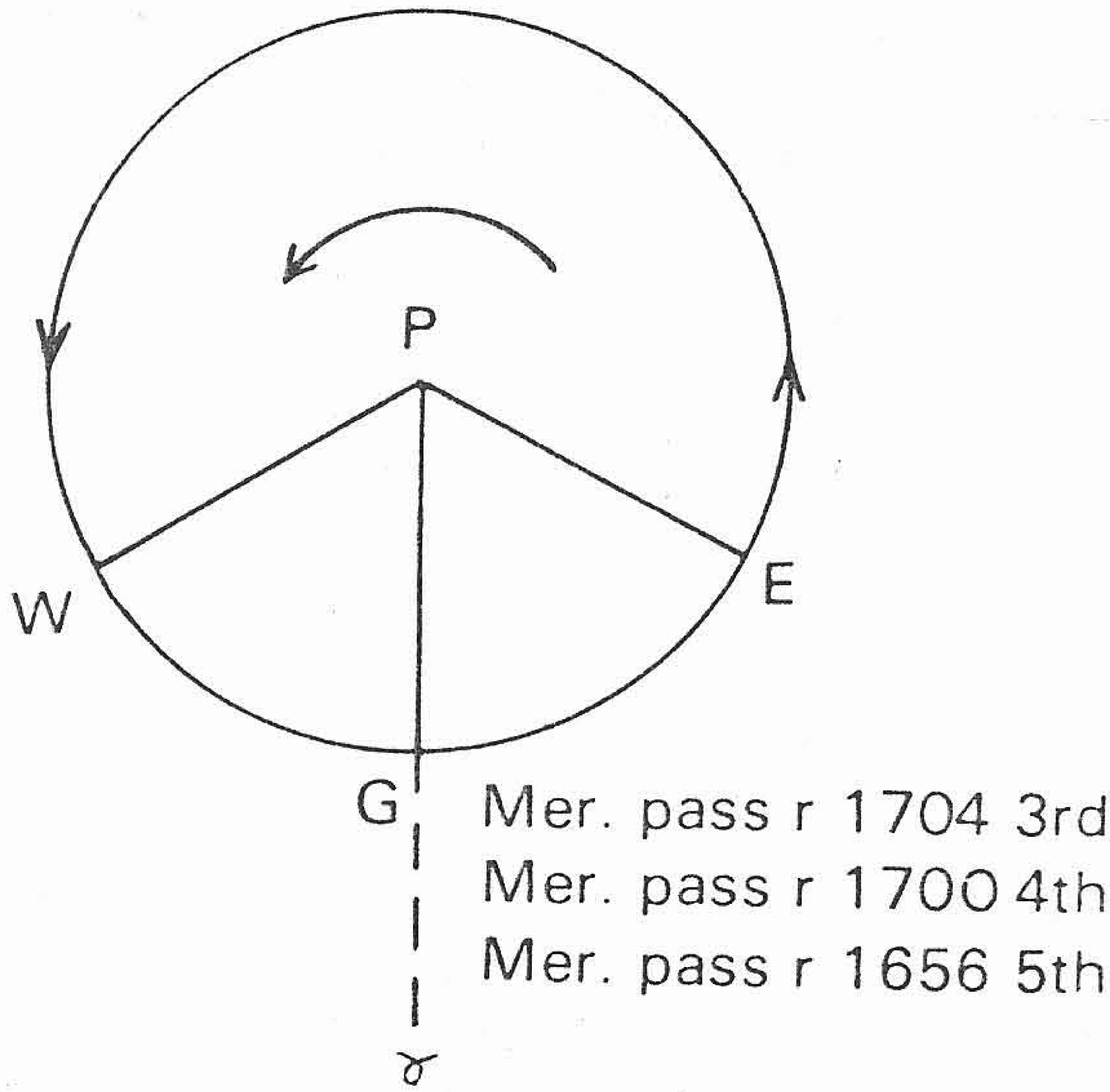


Fig. 53

In fig. 53

PW - meridian of an observer in W. Long
PG - meridian of an observer in Gr. Meridian
PE - meridian of an observer in E. Long.

Say on a certain day, mer. pass. of ♈ at G was 1700 hours. The next day it will be 4 minutes earlier ie. at 16.56 hours. In this interval of one day, the earth has rotated on its axis once and has brought all 360° of Longitudes under ♈ . The earth rotates in the direction of arrow shown in Fig. 53.

An observer on Long. PE will come under ♈ before PG comes under ♈ . Likewise, an observer on PW will come under ♈ after PG has gone past in rotation. In a complete rotation ♈ gains four minutes in time ie. in rotating through 360° ♈ gains 4 minutes. Thus depending on which longitude the observer is,the ♈ would gain proportionately.
Assume Long. PW represents 60° W. Long
and PE represents 60° E. Long

Proportionate gain $\frac{4 \times 60}{360} = \frac{2}{3}$ minute

This proportionate gain is called the **Longitude Correction**. For Easterly longitudes, this correction is + ve, and Westerly longitudes it is – ve because ♈ is gaining on time

LMT Mer. Pass. ♈ at G	= 1700 Hrs	4th
Long Corr. for W. Long	= $-\frac{2}{3}$ min	
LMT mer. pass. ♈ at 60° W	= $1659\frac{1}{3}$	4th

G.M.T. mer. pass. ♈ at G	= 1700	4th
Long Corr. for E. Long.	= $+\frac{2}{3}$ min.	
LMT mer. pass. ♈ at 60° E.	= $1700\frac{2}{3}$ on 4th	

It is evident from this that for observers on Easterly Longitudes the meridian passage time lies between its previous passage and the current date's passages at Greenwich and for Westerly Longitudes, it lies between the current date's passage and the following date's passage at Greenwich.

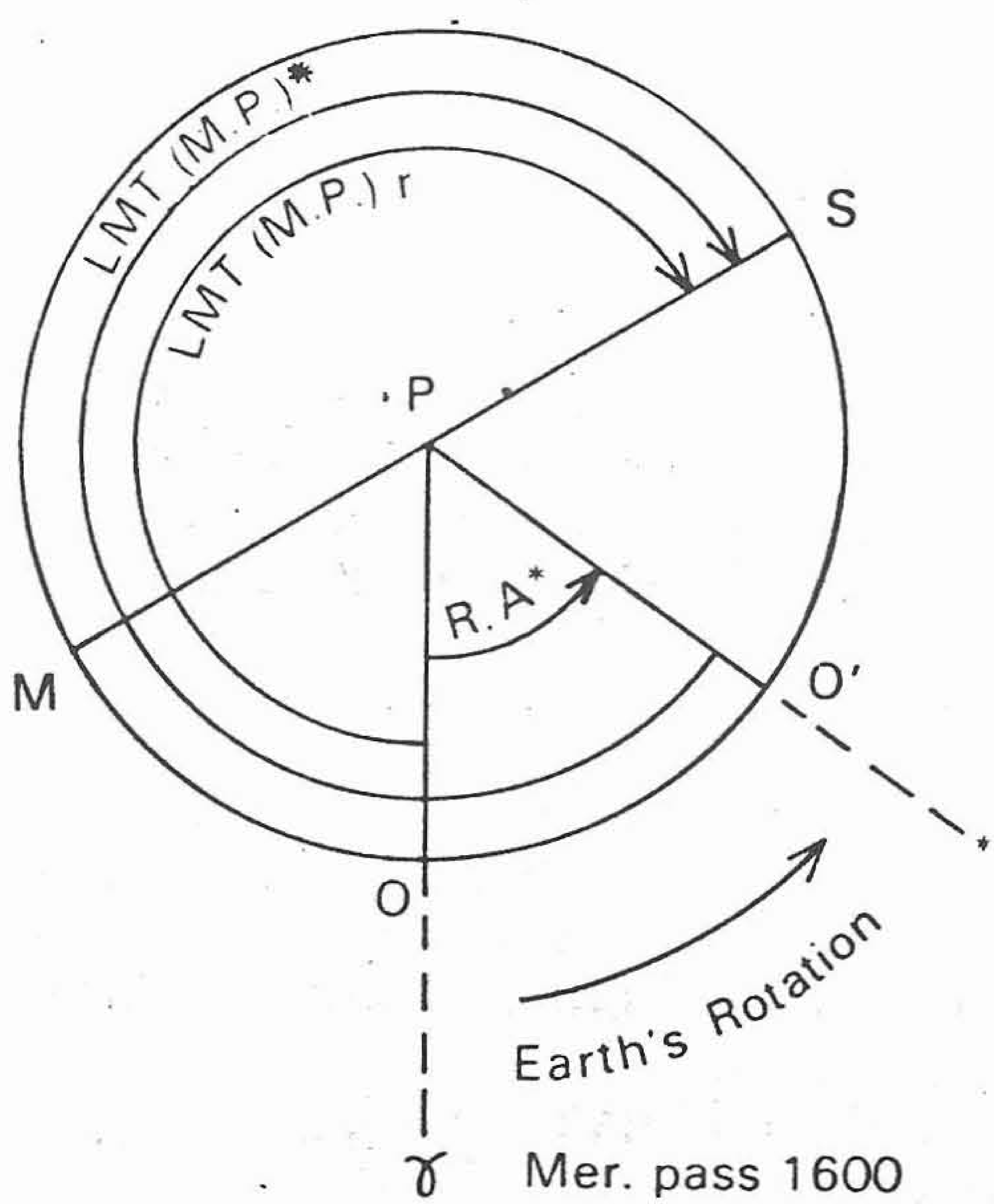


Fig. 54

In fig. 54

PO .. Observer's meridian at transit of ♉
PO' - Observer's meridian at transit of star
PM - meridian of Mean Sun
PS - Meridian of point opp. to Mean Sun
Arc OO' = $O\hat{P}O'$ - R.A. Star.

To show LMT mer. pass. ♉ + RA star = LMT mer. pass. of star.

Let LMT meridian passage of ♉ be at 1600 Hours. Since ♉ is on observer's meridian at 1600 Hours arc OMS measures the LMT at that instant.

After earth rotates through the angle $O\hat{P}O'$ which is equal to RA, the observer is brought to O' with the star on his meridian.

Arc O'O MS is the LMT at that instant

∴ LMT Mer. pass. ♉ + RA star = LMT mer. pass. of star.

Even though the above formula is basically true an additional correction called 'RA' Correction has to be applied in order to obtain the correct LMT meridian passage of the Star. As can be seen from fig. 55, when the star is on observer's meridian PO' the first point of ♉ is on a meridian PL which is further to the West of the observer by an amount equal to the RA of the Star. During the period of rotation of earth, through the Arc RA, ♉, which was on the observer's meridian earlier has been gaining in time.

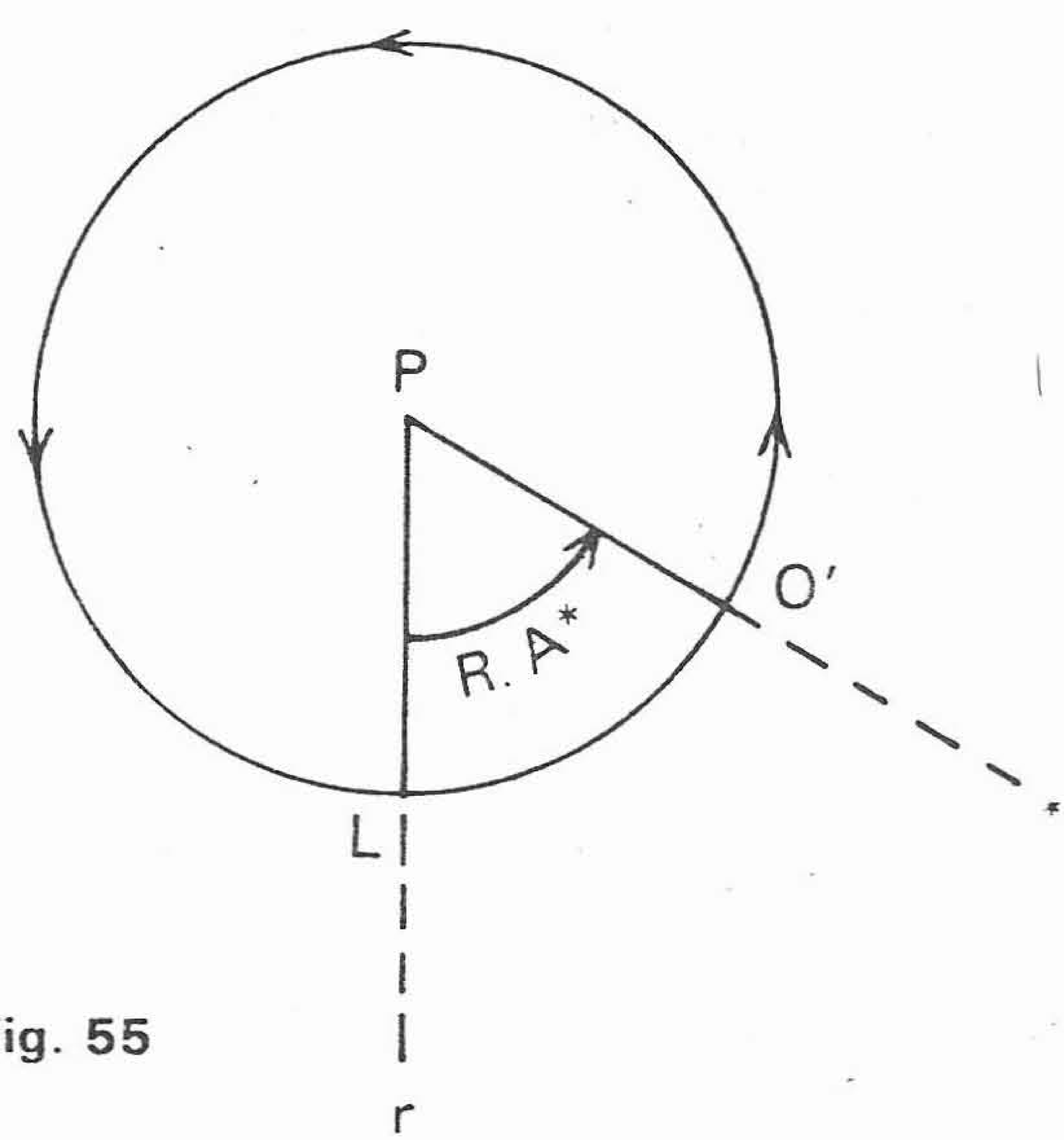


Fig. 55

Intially only LMT meridian passage of ♈ for the observer's meridian could have been worked out by applying the Longitude Correction for his longitude. In fact when the star is on observer's meridian, the longitude correction applied earlier should have been for the long of PL. This ommission is now being made up by applying the RA Correction. This correction is identically same as a Westerly Longitude Correction which is always - ve. Hence RA correction is also - ve. It is calculated proportionately for the RA as follows :

Let the RA be 6 Hours

For 24 Hours of RA - ♈ Gains 4 minutes

For 6 Hours of RA - ♈ Gains $\frac{4 \times 6}{24}$ = 1 min.

Subtract this 1 min. from LMT mer. pass. of star obtained earlier to give the correct LMT mer. pass. of Star.

Worked Example :

Find the LMT mer. pass. of Star "Antares" in DR lat 15° 00'N 120° 00 E on 5th March 1976, given the following data from the Nautical Almanac. SHA ✳ 113° 00.6' mer. pass. of ♈ on the 5th March 1976 (middle day of the page) 13h 6.4.m.

SHA ✳ 113° 00.6' = 7 h 32 m 02s.

Therefore RA ✳ = 24th - 7 h 32m 02s = 16h 27m 58s.

Apporoximate calculations :

Approx. LMT. Tr. of ♈ on 5th		= 13h 6.4.m
RA ✳	+	= 16h 28m
Approx LMT Tr. of ✳ 6th :		= 05h 35m

Since the final LMT Transit falls on 6th at ship, we will have to start off with LMT Tr. of ♈ on 4th so that the ultimate transit will fall on 5th March as required in question.

LMT mer. pass. ♈ at G: March 5th	= 13h	06m	24s
Date Correction for one day	=	+ 3m	56s
LMT Mer. Pass ♈ at G: March 4th	= 13h	10m	20s
Long. Corr. for 120° E Long. $=\left(\frac{4 \times 8}{24}\right)$	=	+ 1m	20s
LMT mer. pass ♈ at ship Mar. 4th	= 13h	11m	40s
+ RA ✳	= 16	27m	58s
LMT mer. pass. ✳ at Ship March 5th	= 05h	39m	38s
RA correction $\frac{4 \times 16.5}{24}$	=	- 2m	45s
LMT mer. pass. ✳ at Ship March 5th	= 05h	36m	53s

Precession of Equinoxes :

So far we had considered the first point of Aries to be a fixed point in space. For all practical day to day operations, this assumption is perfectly correct. But over a period of time the first point of Aries has a slow westward motion along the ecliptic by an average amount of 50" of arc or 3 seconds of time. This slow westward motion of the first point of Aries is called the Precession of Equinoxes.

The earth is shaped like a spheroid, the equitorial diameter being larger than the polar diameter. The earth also keeps spinning on its axis pointing in a N/S direction. The axis is tilted at an angle of about 23½° to a plane at right angles to its orbit. ie. the N. pole of the earth is about 23½° away from the pole of the ecliptic.

The earth behaves exactly like a free gyroscope, and exhibits the two fundamental properties of a free gyroscope viz :

(1) Gyroscopic Inertia (ie. Rigidity in space)

(2) Gyroscopic Precession.

The Sun and the Moon, attract the bulging Zone at the Equator more than they do at the Poles. This difference in attraction tends to pull the equator in alignment with the ecliptic or in other words, tends to make the axis of the earth coincide with a plane at right angles to its orbit. But the earth, due to its gyroscopic inertia, resists this tendency, but instead the pole of the earth makes small circles of radius about 23½° around the pole of the ecliptic.

This behaviour of the earth can be compared with the action of a spinning top, if an external force is applied to the axis of spin when rotating. The point of the top makes small circles around the original point about which it was spinning.

The effect of the change of position of earth's poles is to change the equinoctial correspondingly thereby causing the first point of Aries to shift westward along the ecliptic. This effect is illustrated in the following diagram.

Outer circle represents the Ecliptic. Inner circle represents the path of the earth's poles. P_1 & P_2 are positions of earth's poles at an interval of a few thousand years.

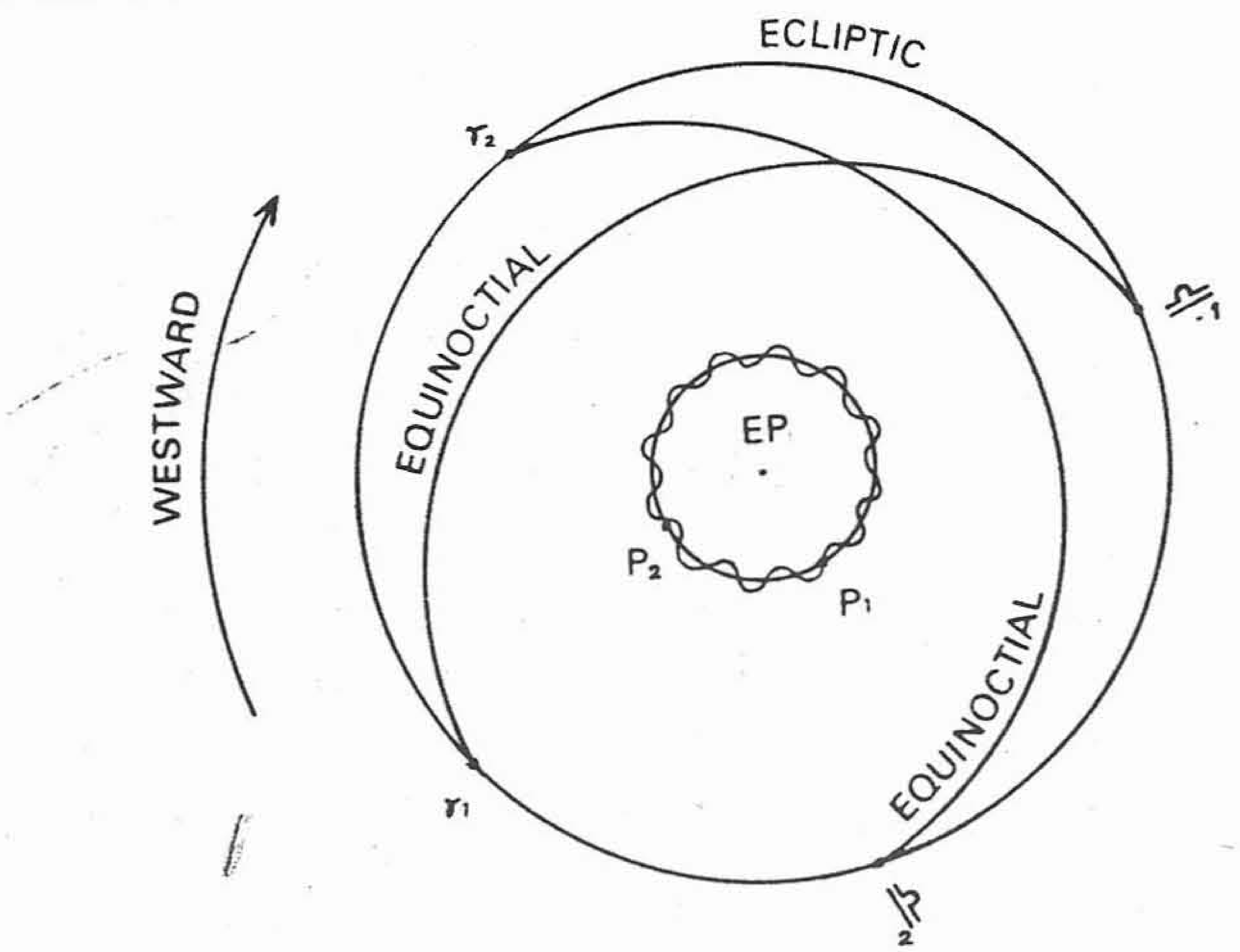


Fig. 56

♈$_1$ ♎$_1$ reps Equinoctial corresponding to P_1 (90° away from P_1)
♈$_2$ ♎$_2$ reps Equinoctial corresponding to P_2 (90° away from P_2)
EP reps Ecliptic pole,90° from Ecliptic.

Wavy line is the modified path of the earth's pole due to nutation. The time taken by the pole of the earth to complete one revolution through 360° round the pole of the ecliptic is about 25800 years. Due to this, after about 16000 years, star Vege will become the Pole star.

The direct effects of Precession are :

(1) It changes the Declination of all fixed stars.
(2) R.A. of all fixed bodies increases or SHA decreases.

Left to Luni-Solar precession alone, these changes may be uniform. But in fact, the changes are not uniform each year due to nutation.

Nutation :

Though the attraction of Sun and Moon is the primary cause of precession, the planets and to a lesser extent the stars that suround the earth also attract the bulging zone at the equator more than they do at the Poles. But this attraction, however small it may be, is uneven. This uneven attraction makes the earth's axis wobble a little as it goes round the pole of the ecliptic, thus producing a wavy path for the poles as shown in fig. 56. This effect is called **Nutation.**

The direct effects of Nutation are that it causes minute changes in the obliquity of the ecliptic & it makes the precession itself uneven.

The combined effects of Precession and Nutation are :

(1) The change in RA or SHA of fixed stars are uneven each year.
(2) The changes in Declination of fixed bodies are also uneven.
The indirect effect of Precession is that it causes a difference between a Sidereal year and the Tropical year.

Sidereal Year : is the time taken by the sun to go 360° round the earth once,starting in line with a star, back, in line with the same star. This period is 365.256 days (356 days 6 h 9m 9s).

Tropical Year : is the time taken by the sun to go round the earth once, starting from first point of Aries back again to first point of Aries. Due to precession, since Aries itself has moved nearly 50" of arc to the westward in the interval, the length of a tropical year is slightly shorter than a sidereal year. The Tropical year is 365.2422 days (365 days 5h 48m 46s). The Civil year is based on the Tropical year. If not, the seasons will not recur at the same time each year.

The anomalistic year is the time taken by the earth to go from perihelion to the next perihelion and is equal to 365 days 6h 13m 53s.

Civil Calender : It is interesting to note how a Civil year is adjusted to equal the Tropical year. Of necessity, the year can only have whole number of days. Hence a Civil year has 365 days.The short fall of0.24of a day each year adds up to nearly one day, which is added as one extra day in February, every fourth year, which is a **leap year.** But this would be adding too much by 001 of a day each year. So this one day is removed every 100 years. Hence completion of most centuries are not leap years. To know whether completion of a centuary is a leap year or not the number ommitting last two zeros, should be exactly divisible by four, eq. 1700, 1800, 1900, 2100 are not leap years. 1600, 2000, 2400 are leap years. Every 4th centuary becomes a leap year because, by taking a year as 365.24 days 0.0022 days are missed out each year. To make up for this, one day is added on every 400 years. The residual even after all these adjustments amounts to a day in nearly every 4000 years.

Exercise VIII

(1) Why does a star come on the meridian four minutes earlier each day ? Explain and illustrate.

(2) Convert 10h 18m 50s of Mean Solar time into its corresponding Sidereal time.

(3) Convert 6h 26m 22s of Sidereal time into its corresponding Mean Solar time.

(4) Prove with the aid of an appropriate diagram that :-
(a) GHA ♈ + SHA * – W. Long = LHA *
(b) LMT mer. pass. ♈ + RA * = LMT mer. pass. *

(5) Find LMT mer. pass. of * Sirius at Ship in lat. 10° 00′ N 60° W on 10th October 1976 and find also the corresponding GMT given that LMT mer. pass. ♈ at Greenwich for the middle day of the page in the Almanac 10th Oct. was 22h 41.4m and the SHA * Sirius was 258° 58′.

(6) Explain what is Precession of Equinoxes and what are its direct effects.

(7) What is Nutation ? What effect has this on Precession of Equiroxes

(8) Define the following terms : Sidereal day, sidereal year, Tropical year, Anomalistic year.

(9) Why is it necessary to have the basis for a Civil year as the Tropical year ? Why not base it on Sidereal Year ?

Answers :

Q. 2. 10h 20m 31.7s.

Q. 3. 6h 25m 18.7s.

Q. 6. LMT Oct 10d 05h 27m 47s.

GMT Oct 10d 09h 27m 47s.

CHAPTER IX -

THE EARTH - MOON SYSTEM

The Moon is the only natural satellite earth has. The moon moves round the earth in an elliptical orbit and follows the kepler's Laws of Planetary motion like any other planet. The eccentricity of the moon's orbit is slightly greater than that of the earth. In its orbit, when the moon is closest to the earth it is said to be in **perigee** and when it is farthest away, it is said to be in **apogee.** The line joining apogee to pergee is called the **Line of Apsides.** The mean distance between Moon and earth is 239,000 miles. The variation in distance changes the semi-diameter of the moon from 14.7′ of arc at apogee to 16.4′ of arc at perigee.

Sidereal period of the moon, is the time taken by the moon to go round the earth once commencing from a position in line with a star and come back again in line with that star after having covered 360° in its orbit. In terms of mean Solar time, this period is equal to $27^1/_3$ days (27d 7h 43m 11.5s to be exact). This gives an average rate of motion of the moon on its orbit as $\frac{360}{27.33}$ =13.17°per day. This daily motion is much more than any other astronomical celestial body. Since this motion is direct, the SHA of Moon will continuously decrease at the rate quoted above.

When we consider the movement of the moon in relation to the Earth-Sun System, the perspective somewhat differs.

Consider the earth in position 'A' (in fig. 57). The moon 'M' is in conjuction with the Sun (S) and a distant star X. When earth has moved to position 'B' after one sidereal period, the Moon at M, is in line with Star X, but it is not in conjuction with the Sun. It will take another day or two for the Moon to move through arc M, to M_2 to be in conjunction with the Sun again.

This period from one conjunction with Sun to the next conjunction with the Sun is called a **"Synodic period"** or a **"Lunation"**. It is sometimes also referred to as a **"Lunar month"**. The length of a "Lunation" is not constant. It has an average value of 29½ days ie. 29d 12h 44m 3s and has a variation of ± 12 hours.

Perihelion
*X
*X
D
C
E
E
M_4
M_5(Conj)
M_3 (Conj)
S
M (Conjuction)
M_2 (Conj)
M_1
E
E
A
B
Y*
Y*
Aphelion

Fig. 57

The reason why the length of a Lunation varies can be seen when we compare the movements of the earth at aphelion and perihelion.

In figures 57 when the earth is in perihelion at position 'C' let M_3 be the position of Moon, in conjuction with Sun (S) and in line with a distant Star 'Y'. After the completion of a sidereal period when earth has reached position 'D', the moon reaches position M_4 in line with Star 'Y', but it is not in conjunction with Sun. It has to move a further arc M_4 to M_5 before conjuction can occur.

Compare this exceess arc M_4 to M_5 with arc M_1 to M_2 at aphelion. Since the earth has moved through a larger arc in perihelion, the moon also has to move a larger arc M_4 to M_5 to complete the conjunction.

Thus length of Lunation is greater at perihelion than at aphelion. At any other part of the orbit its length will be somewhat between the two extremes.

The plane of the moon's orbit is inclined to the ecliptic at an angle of 5¼° The points at which these two planes cross are called the **Nodes.** That point of crossing at which the moon is moving from South to North declination, it is called the **'Ascending Node'** and that point diametrically opposites, ie. the point of crossing at which the moon is moving North to South is called the **Descending Node'.**

Like the first point of Aries and Libra which precess along the ecliptic each year, the moon's Nodes also precess westward along the ecliptic each year taking 18.6 years to complete one revolution round the ecliptic ie. its annual regression is about 19°,

As a result of this motion the Moon's maximum declination changes year to year and oscillates between 23½° + 5¼° = 28¾° N or S and 23½° - 5¼° = 18¼° N or S during one half cycle of the regression of Nodes.

For example, in a year when the ascending Node coincides with first point of Aries, the maximum declination of moon will be 23½° + 5¼° = 28¾°N or S. After 9.3 years, when the ascending Node has regressed through 180° ie. when it is coincident with first point of Libra, the maximum declination of moon that year will be 23½° - 5¼° = 18¼°N or S. During the next 9.3 years, the maximum diclination of the moon will again gradually increase to 28¾°N/S when the ascending Node coincides with first point of Aries. This is illustrated in figures 58 (A) & (B).

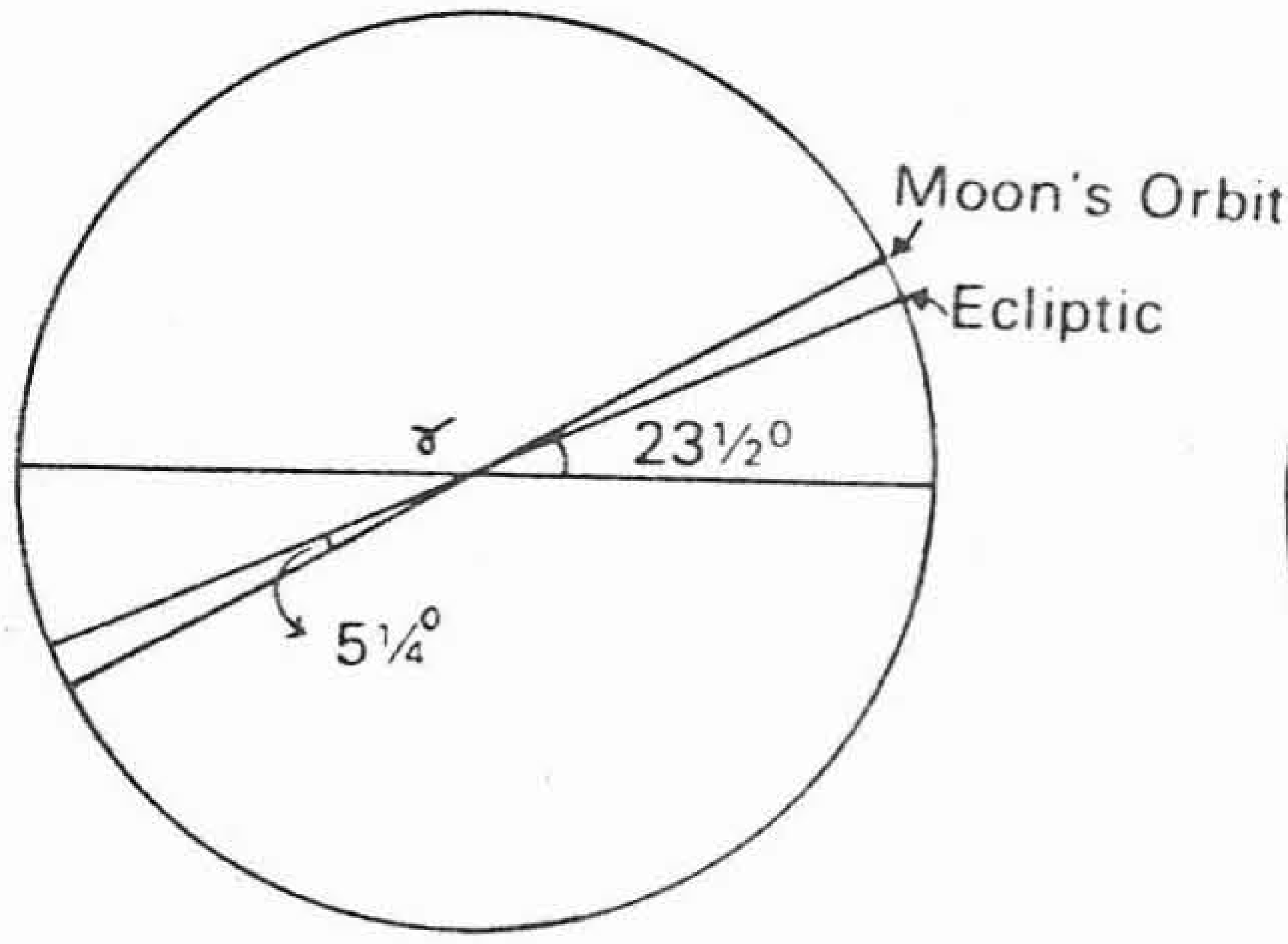


Fig. 58 (A)

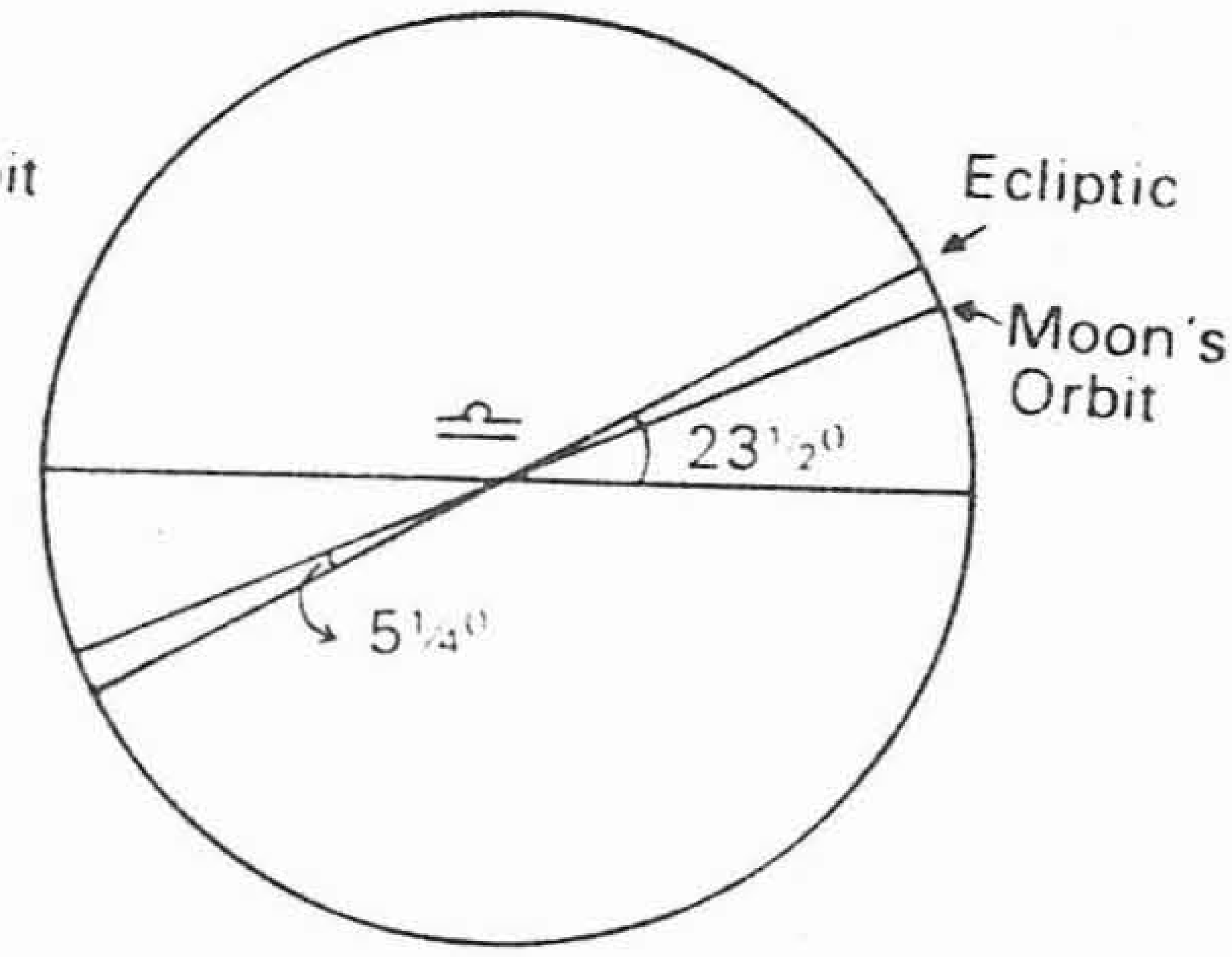
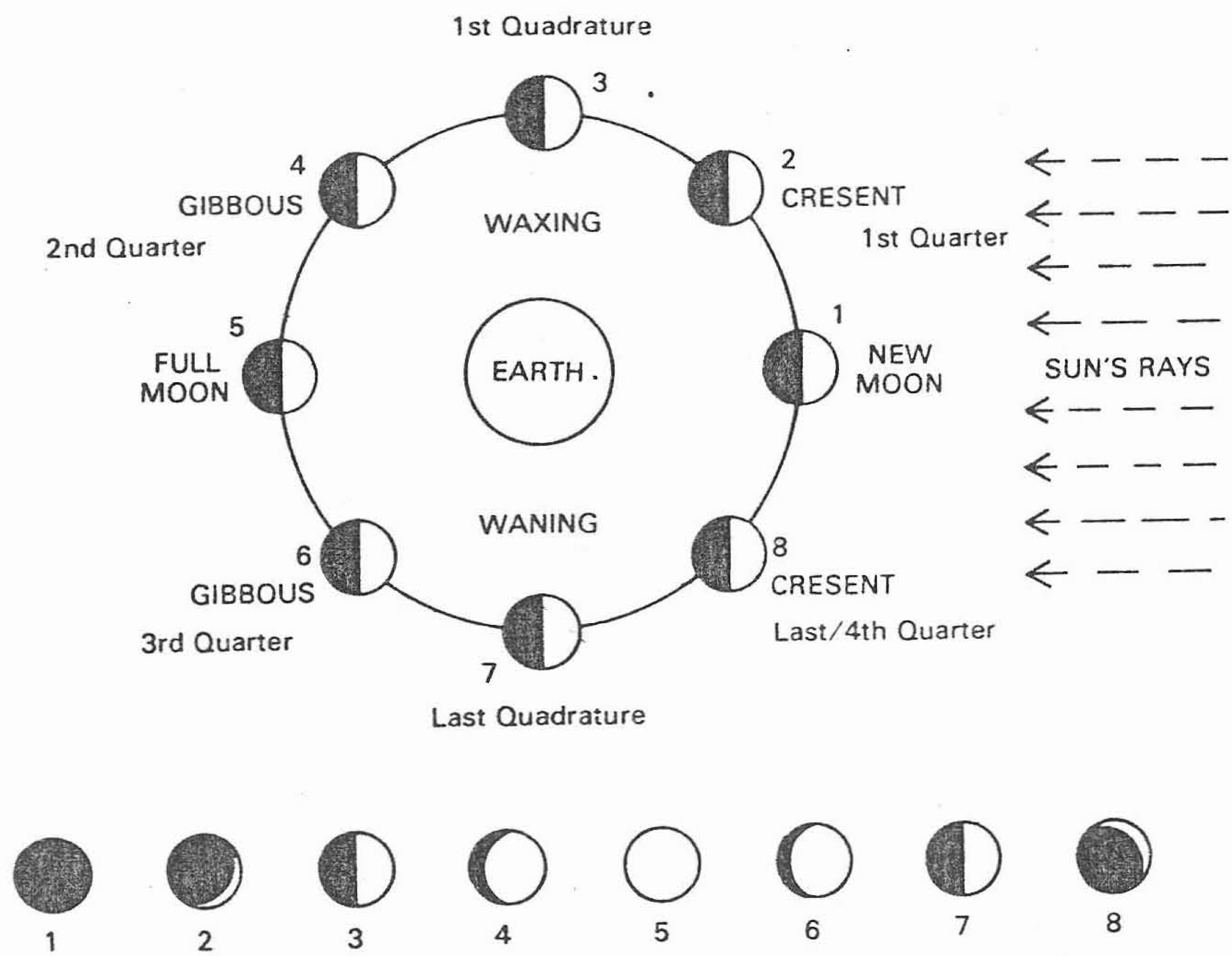


Fig. 58 (B)

Phases of the Moon :

During one Lunation, the moon presents differing arcs of its lighted surface to the earth each day. This phenomenon is called the **phase of the Moon.**

Unlike the Sun, the Moon does not shine by its own light. It merely reflects the light received from the Sun. At any time half the surface of the moon is lit by Sun's rays falling on its surface. Fig. 59 represents eight positions of the moon during one Lunation. In position 1, the entire lighted surface is turned away from the earth. The moon is said to be newly born or **New Moon.** The day is called **New Moon day.** Sun and Moon are both on the same meridian and on the same side of the earth. They are then said to in **"conjunction".** The age of the moon then is zero. Moon and Sun rise, cross the meridian and set together on this day. Day after day, the moon moves on its orbit. A few days later when it reaches position 2 even though half the surface of the moon is still lit by Sun's rays only a small arc of the lighted surface is visible from the earth and it then appears as a **cresent moon.** Each day this lighted arc increases and when it has covered quarter of its journey and reaches positon 3, half the moon appears to be lit as seen from the earth. The moon is said to be in **First Quadrature** with the Sun ie. Sun and moon making about 90° angle at the earth. The age of the moon on that day is about 7½ days. Exactly half moon is visible from the earth and



PHASES OF THE MOON

Fig. 59

the moon is then said to be dichotomised. During the period between New Moon and 1st Quadrature, the moon is said to be in its First Quarter. As the Moon progresses on its orbit, the lighted arc presented to the earth still keeps on increasing. In position 4 when more than half surface appears to be lighted, it is said to be a **Gibbous moon.** Nearly 15 days after new moon, it reaches position 5, when its full lighted surface can be seen from the earth. The Moon and Sun are then said to be in **Opposition** ie. on opposite sides of the earth. The age of the moon then is about 15 days. On this **Fullmoon** day, when sun rises, moon will set and vice versa; they being on opposite meridians then.

During the period New Moon to Full Moon, the moon is said to be waxing ie. growing bigger each day. Moon is said to be in its Second Quarter, between the day of 1st Quadrature and Full Moon day.

During the subsequent days, following the full moon, the lighted surface presented to the earth, keeps on decreasing. In position 6 during the Third Quarter of the orbit, it again appears as a gibbous moon.

When it reaches position 7, where Sun and Moon are again at 90°, to the earth the moon is said to be in its last Quadrature and only half the lighted surface is been from the earth. The age of the moon then is about 22½ days.

From position 7, to position 1, through position 8, the phase gets smaller and smaller into a cresent form. During this period the moon is said to be in the IVth or Last Quarter.

From the new moon in position 1, when the age is nearly 30 days, on completion of the Lunation, the cycle repeates again. It will be seen from the above explanation that the **age of the moon** is the number of days elapsed after the last new Moon.

The part of the moon facing the earth, but not illuminated by Sun's rays, is often visible as a dim glow. This is due to the light reflected from the earth falling on the moon.

If an observer, were to be situated on the moon, and sees the earth; to him it would appear as if the earth is presenting similar phases as the moon, except that it would be in opposite phase with that of the moon ie. when it is New Moon earth would appear Full and vice versa.

The Moon's Rotation : Similar to earth, the moon also rotates on its axis. The Moon's axis is tilted at an angle of 6½° from a plane at right angles to its orbit, unlike the earth whose tilt from the perpendicular plane is 23½°. The time taken by the moon to rotate on its axis once and its sidereal period is the same. This is evident from the fact that the moon presents the same face to the earth always. The rotational period and the sidereal period have become synchronous, because of gradual slowing down of speed of rotation through millions of years, due to the friction between the tidal wave and the body itself rotating under the tidal wave. This gradual slowing down continues till such time as there is no rotation of the body relative to the tidal wave. In fact, the same phenomenon is likely to take place in the satellites of other planets as well. Even the earth is gradually slowing down its rotation, which accounts for the lengthening of a day by 0.002 seconds is a centuary.

Because the same face of the moon is turned towards the earth, always an observer on the earth can see only 49% of the Moon's surface at any one time. However, some time or the other, additional surface area of the moon has been observed from the earth, provided the Moon and Sun are suitably situated. Whenever the additional area on the fringes of the lighted surface is seen, the Moon is said to be **"Liberated"**. The phenomenon is called **Liberation.**

Liberation in Latitude : Since the axis of the rotation of Moon is tilted at an angle of 6½° to the perpendicular, some times the moon's north polar region or south polar region is seen from the earth, depending of which is tilted towards the earth. The declination of Sun and Moon do play a part in it viz : if the moon has a high north declination and the sun has a high south declination, then the South pole of the moon can be seen. If the declinations are other way round, the North polar regions will be seen. This is Liberation in Latitude.

Liberation in Longitude : Moon's orbit being eliptical its angular velocity along the orbit varies, following Keplar's laws of Motion. Hence even though it has a constant speed of rotation on its axis, which sychronises with its sidereal period, the speed of orbital revolution gets slightly out of step with the axial rotational speed, sometimes leading or sometimes laging. During these times, it is possible to see a small additional area of the moon along the Eastern or Western edges of the lighted surface and this is termed the Liberation in Longitude.

Taking all these factors into account, a total of 59% of the surface area of the moon has been observed from the earth though not all at the same time. The remaining 41% has never been seen directly from the earth. The only way to know that surface is from photographs taken of that side by space probes.

Delay in the meridian passage of the Moon each day.

We are keeping time by the Sun. Hence, any delay or gain is reckoned with respect to the Sun. Referring to Fig. 60, assume on any one day the Sun (S) and the Moon (M) are on the meridian (PO) of the observer (O). After an interval of 24 hours, when the observer has been rotated on the axis once and the Sun is again

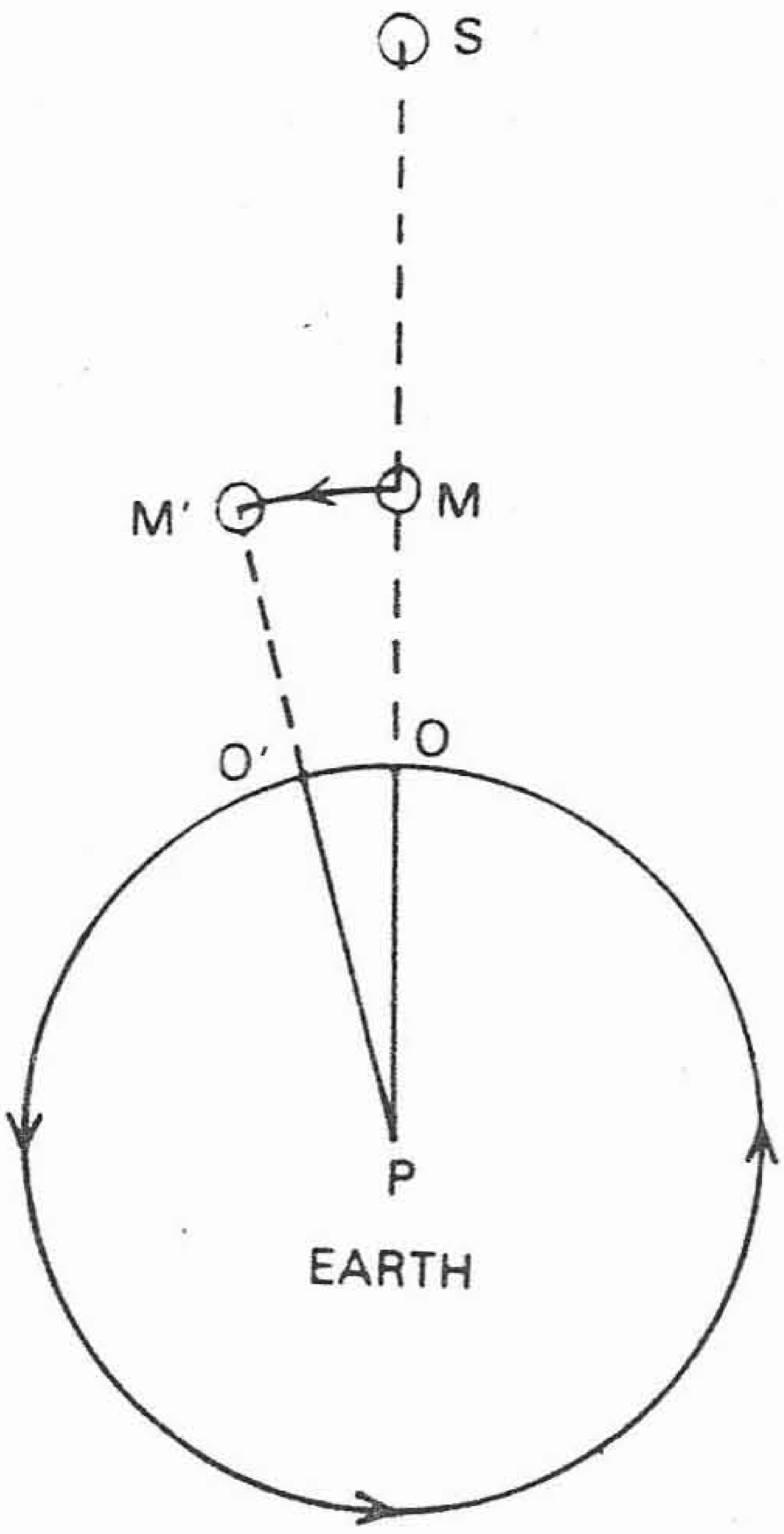


Fig. 60

on the meridian, the moon is no longer at M. It has moved to new position M′ because of its orbital motion. We saw earlier in this chapter that the daily motion is about 13.17° per day. Thus the observer has to be rotated through this additional arc before he can get the moon on the meridian. This is just under one hour. The average delay is taken as about 50 minutes of mean solar time. Since the rate of orbital motion of the moon along the elliptical orbit is dependent on kepler's second law of motion, the actual arc the moon moves on its orbit varies, being least in apogee and greatest at perigee. The delay can therefore be anywhere between 40 minute and 63 minutes. Thus we see that the moon comes on the meridian approximately 50 minutes later each day. The exact delay for any given day is obtained by comparing the meridian passage time of the moon, tabulated for each day in the nautical almanac.

Moon Rise and Moon Set : For the same reason, as stated above there will also be a corresponding retardation in Moon rise and Moon set. Times of Moon rise and set are further modified by the effects of changes in the declination of the moon day to day coupled with the latitude of the observer.

The effect of change of declination is greatest in high latitudes. In extreme cases, the moon appears to rise or set earlier each day or even some times rise twice on the same day, thus offsetting completely the normal retardation due to its orbital motion. The times of the moon rise and moon set, tabulated for each day in the nautical almanac is illustrative of this fact.

Eclipses :

The terms Eclipse is used to denote that particular phenomenon whereby the Sun's face appears to be covered by the Moon coming between the earth and the Sun, or when the earth obstructs the Sun's rays from reaching the moon, by coming between the Sun and Moon. The former is called **Solar Eclipse** and the latter is called **Lunar Eclipse.**

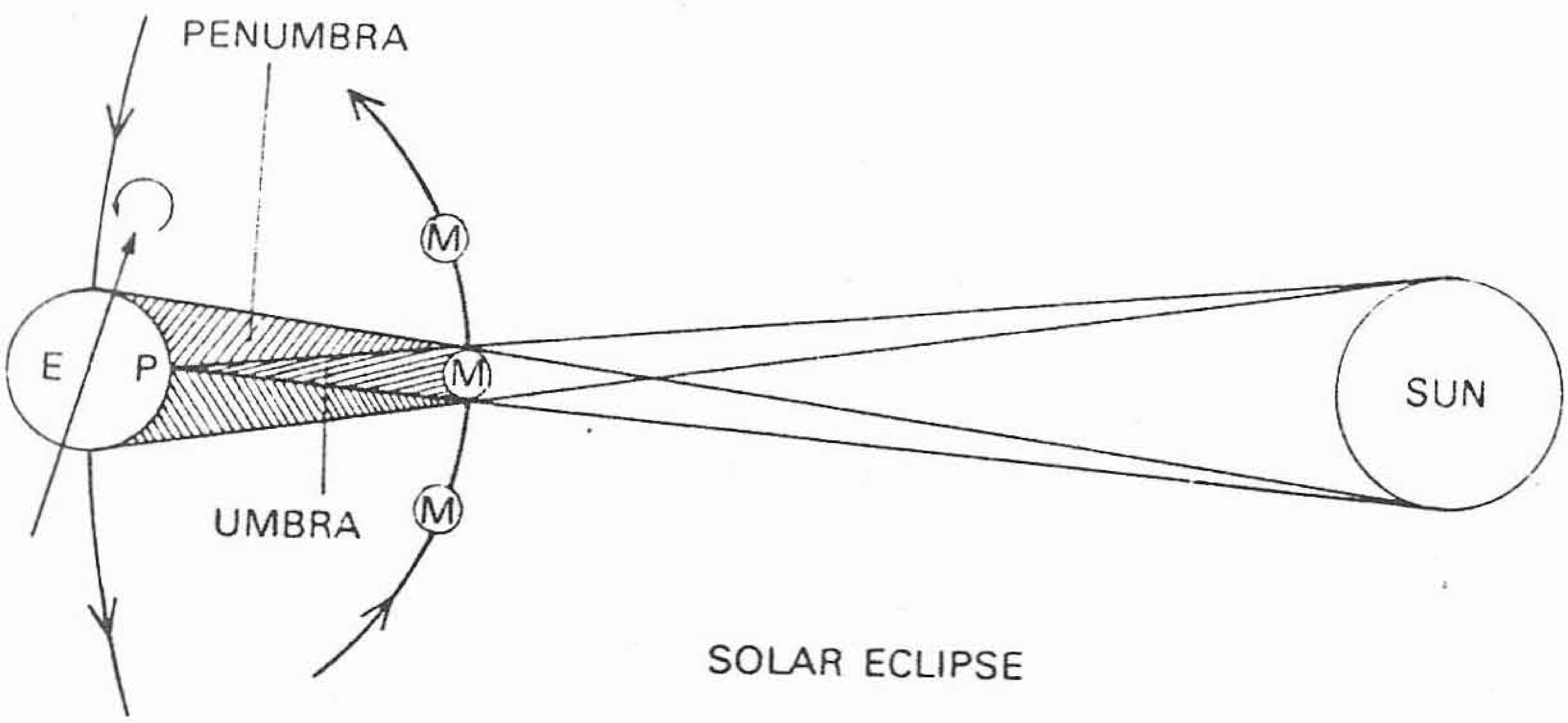


Fig. 61

Solar Eclipse : These are of three types (1) Total Eclipse (2) Annualar Eclipse (3) Partical Eclipse.

Solar Eclipse occurs when the Sun, Moon and Earth are in one direct line,with the moon interposed between Sun and the Earth. The moon casts a dark conical shadow called **"Umbra"** which strikes the earth over some area 'P' Fig. 61. On each side of the Umbra there also exists areas of partial shadow called **"Penumbra"**.

Observers directly below the Umbra will see the entire face of the sun covered by Moon and will then be experiencing a **Total Eclipse.** The observers in the surrounding areas where only penumbra reaches, will only see a part of the sun covered and they will be experiencing a **Partial Eclipse** of the Sun.

The moon being a small body, the length of the umbra is just enough to reach the earth when the moon happens to be in perigee. But if the eclipse occurs when, moon is in apogee the tip of the umbra stops short of the earth and umbra gets divided and observers directly below the divided umbra see the entire diameter of the moon. within the face of the sun, and there is a ring of Sun's disc outside the Moon's disc. Such an eclipse is called an **Annular Eclipse.**

For a Solar Eclipse to occur, the conditions necessary are :

(1) The Sun and Moon must be in conjunction which can happen only on a new Moon day.

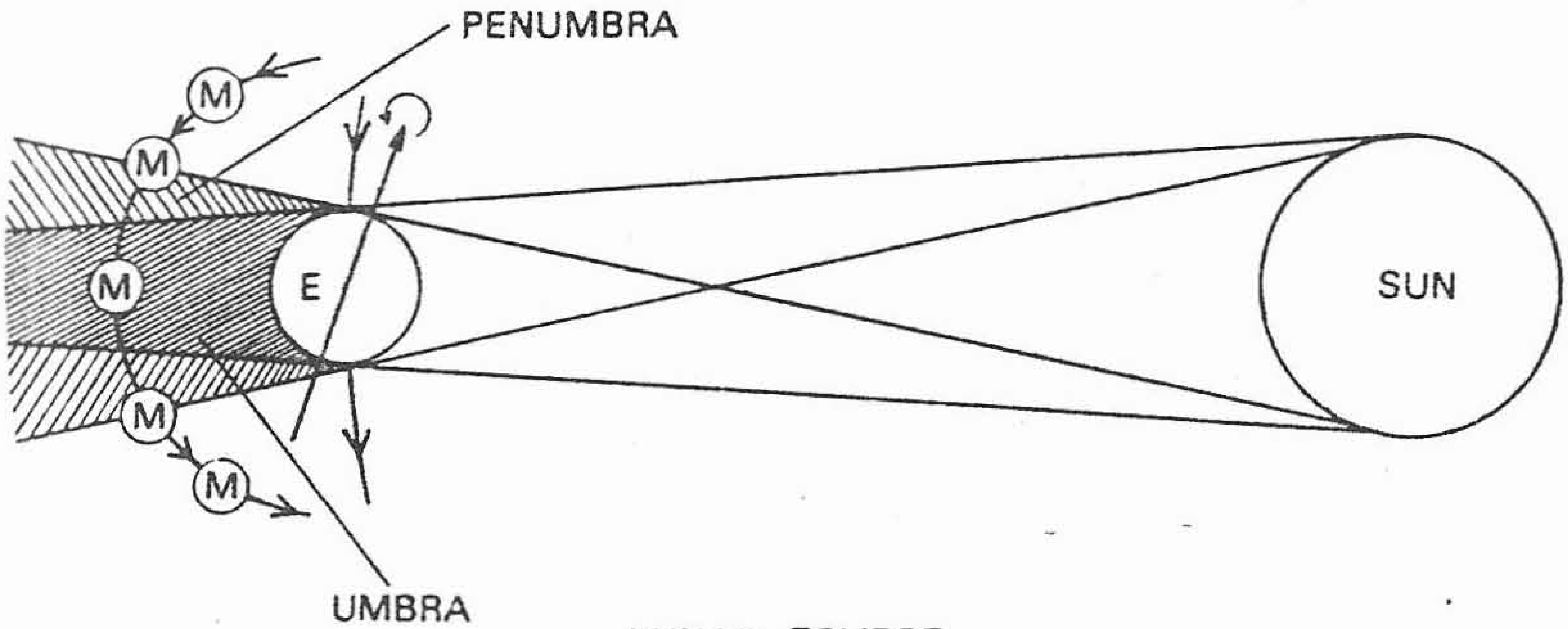
(2) In order that all three will be in a direct line, the geographical position of Sun and Moon must be the same ie. their declinations and RA or SHA or GHA must be the same.

At every new Moon, both sun and moon are in conjunction, but it does not develop into a Solar Eclipse because, eventhough their meridians are the same, at conjunction, their declinations are not the same every time. This is because of the fact that the moon's and earth's orbits are not co-planer, ie. on the same plane. Since the moon's orbit makes on angle of about $5\frac{1}{4}^{\circ}$ with the earth's orbit, the declinations of the sun and moon will be same only if conjunction takes place at or near the nodes of the Moon's orbit.

From Fig. 61, it will be observed that during the eclipse, the orbital motion of moon and earth are in opposite directions as shown by the arrows on their respective orbits. So they are passing each other at a relative speed equal to the sum of their orbital rates of motion. Hence, the duration of the totality of a Solar eclipse is very short. Further, during the totality since the earth continues to spin on its axis. So, the point of the cone of the umbra, traces out a path along the earth's surface. The path so traced out is called the **Path of the Eclipse.** Since only the tip of the cone reaches the earth, covering only a very small area, coupled with the fact that the totality is also very short lived, only very few observers situated along the path of the eclipse can observe the total eclipse of the Sun. Other areas will only see it as a partial eclipse. Hence it appears for a stationary observer on the earth, that Solar eclipses are few and far between which in fact is not so.

Lunar Eclipses are of two types (1) Total Eclipse (2) Partial Eclipse.

Lunar Eclipse occurs when Sun, Moon and Earth are in one direct line, with the Earth interposed between the Sun and Moon. Earth being a much larger body than the Moon, the conical shadow of umbra cast by the earth invariably reaches the Moon, irrespective of whether the moon happens to be in perigee or apogee. On either side of the umbra, a partial shadow of the penumbra region also exists for the earths shadow.



LUNAR ECLIPSE

Fig. 62

In Fig. 62, the arrows show the direction of motion of Moon and Earth in their respective orbits. The Moon's orbital motion being much greater than the earth, it passes through the shadow region and overtakes the earth. When it enters the penumbra, part of the moon begins to get covered by the earth's shadow and when the moon is in the umbra region of the earth's shadow, the entire rays of the sun are cut off from reaching the moon and the moon is then totally covered in the shadow. This is now a **Total eclipse.**

For a Lunar Eclipse to occur, the conditions necessary are :-

(1) The Sun and Moon must be in opposition, which can happen only on a full moon day.

(2) In order that they will all be in one direct line the geographical position of Sun & Moon must be diametrically opposite ie. the declination of Sun and Moon must be numerically same, but opposite in name and their RA or SHA or GHA must be 180° apart.

At every full moon, both sun and moon are in opposition and their SHA is 180° apart ie. they are in opposite meridians, but it does not develop into a Lunar eclipse. This is because the orbits of Moon and Earth are not co-planer. Since,the two orbits make an angle of about 5¼° with each other the condition necessary to make the declination of sun & moon suitable for an eclipse will only take place if opposition occurs when sun & moon are at or near the opposite nodes of the moon.

As may be observed from fig. 62, since the moon and earth are moving in the same direction, the rate at which the moon will be passing through the shadow area will be the difference of the two orbital speeds. Further, the earth's shadow region is also large. Hence the moon takes several hours to pass through this zone. The totality of a Lunar Eclipse (the time during which the entire face of the moon remain totally covered) is therefore much longer than the Solar Eclipse. During this period the earth continues to rotate on its axis and brings large number of observers in front of the moon. Thus many observers see a Lunar Eclipse. Only those observers who pass out of the moon's view before totality of the eclipse or those who come into view after totality of the eclipse will see it as a **Partial Eclipse.** Since large number of observer's see this eclipse, it appears as if lunar eclipses are more frequent than solar eclipses, which in fact is not so.

There is no path for Lunar Eclipse, because the umbra of moon does not fall on the earth, but the earth itself is procducing the umbra through which the moon passes.

Number of Eclipses in a year :

If the positions of Sun and moon are very favourable with respect to the Moon's nodes, then there can be as many as seven eclipses in a period of twelve months ie. either five solar and two lunar or four solar and three lunar eclipses. When their positions with respect to the moon's nodes are least favourable, then there can be only two eclipses in a period of twelve months, both of which must be solar eclipse.

Though Solar Eclipses are more frequent than Lunar eclipses at any one place on the earth, Lunar eclipses are seen by large number of observers over a larger area of the earth, whereas the solar eclipses are only seen on a limited area and hence, visible only to a few observers.

Occultation is the term used to denote an occurance similar to Solar eclipse. If the Moon or Sun in its orbital motion around the earth, obstructs the rays of a planet or star which is in conjunction with the luminary from reaching the earth, then that phenomenon is called an **"Occultation"**. For this to occur, the Declination and SHA of Moon or Sun and the farther planet or star must be the same. Occultation by Moon can be observed. Though the Sun can similarly occult a farther star or even a planet in conjunction with the sun, these cannot be observed owing to the brightness of the sun.

Exercise IX

(1) Define the following terms :

Sidereal period of the moon, Lunation, Synodic period of the moon, Quadrature, Gibbous moon, Conjunction, Opposition.

(2) Explain with a sketch the phases of the moon.

(3) Why does the moon come on the meridian approximately 50 minutes later each day ?

(4) State the LAT of moon's meridian passage over any meridian on (a) new moon day (b) full moon day (c) Ist Quadrature (d) Last Quadrature.

(5) Explain with appropriate diagrams : (a) Solar Eclipse (b) Lunar Eclipse ?

(6) What conditions are necessary for : (a) Solar Eclipse ? (b) Lunar Eclipse ?

(7) Why is there no eclipse every full moon and new Moon ?

(8) Why does the moon present the same face to the earth always ?

(9) What do you understand by the term "Liberation" of the Moon ? Distinguish between Liberation in latitude and Liberation in Longitude ?

(10) What is the maximum and minimum number of eclipses that can happen in a year ?

CHAPTER X

RELATIVE MOTION OF PLANETS

In chapter III, it was discussed how the nine planets moving round the sun in well defined orbits made up the solar system. The position of the planets in their respective orbits, in relation to the sun as viewed from the earth, are given special names.

Inferior Planets are those planets whose orbits lie closer to the sun then the earth. These planets are Mercury & Venus.

Superior Planets are those planets whose orbits lie further away from the sun than the earth. These planets are :-
Mars, Jupiter, Saturn Uranus, Neptune & Pluto.

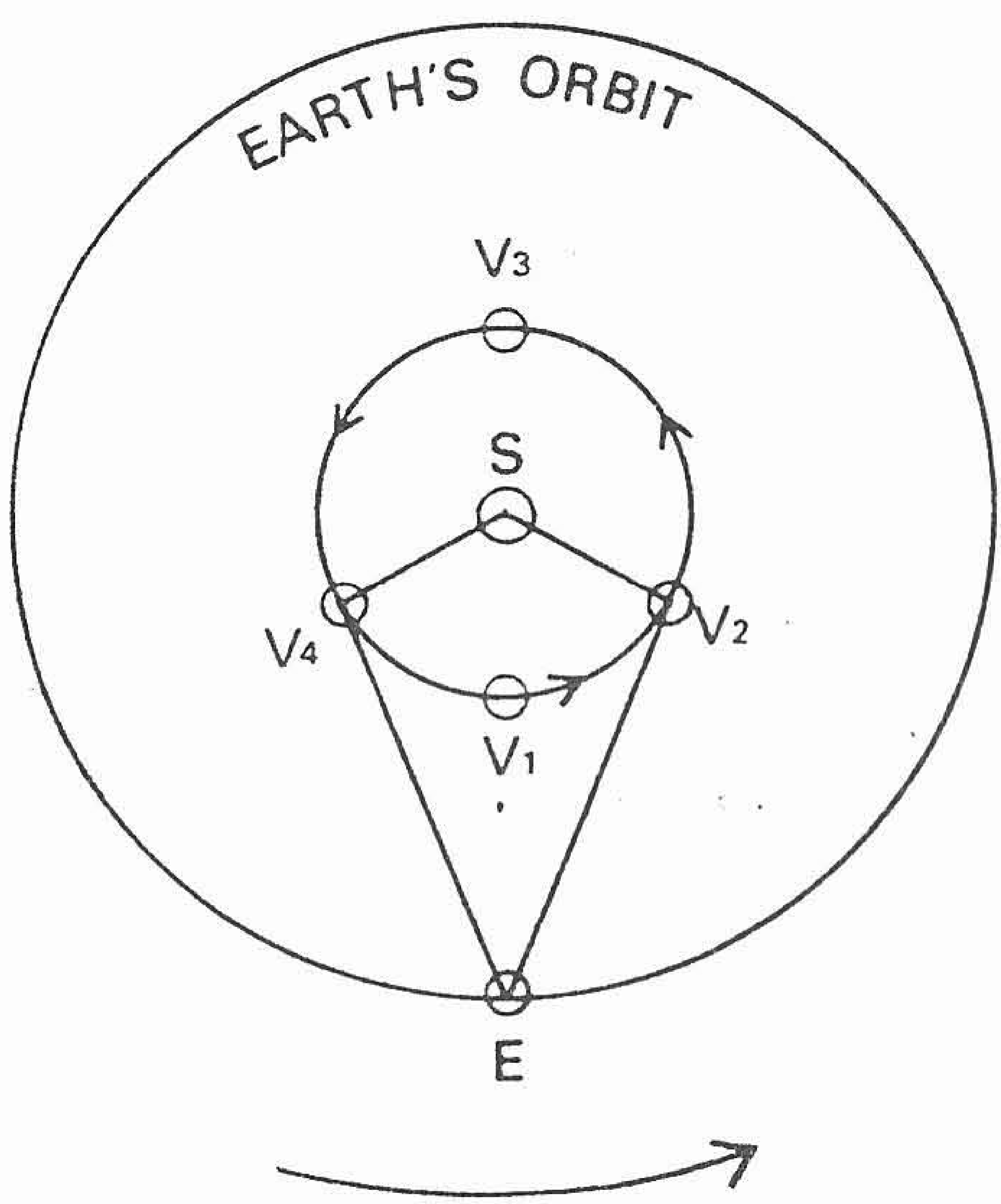


Fig. 63 Inferior planets

Position V_1	Inferior Conjunction
Position V_3	Superior Conjunction
Position V_2	Maximum Elongation (West)
Position V_4	Maximum Elogation (East)

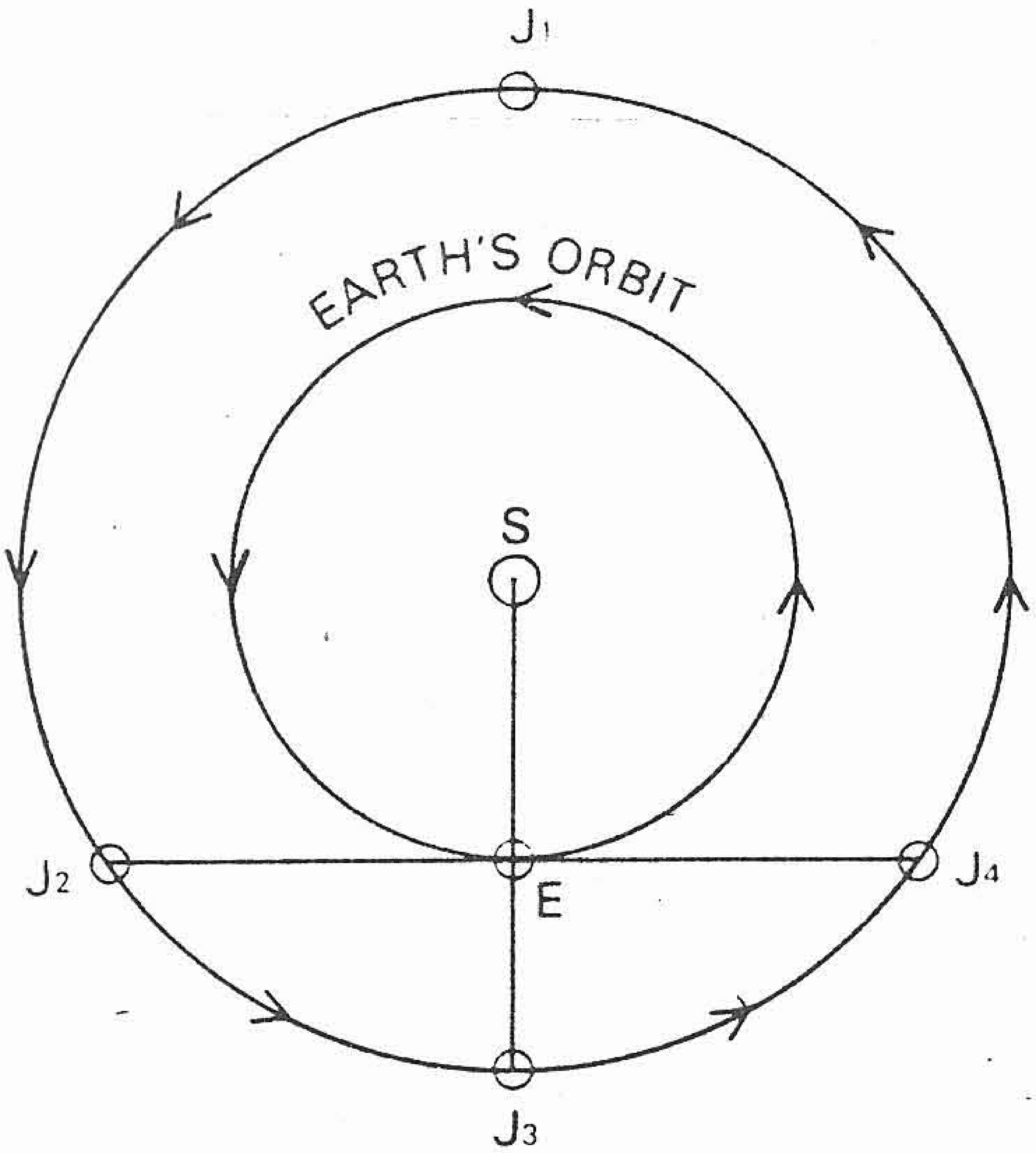


Fig. 64 Superior planets

Position J_1	Conjunction.
Position J_3	Opposition.
J_2 & J_4	Quadrature.

It may be observed from fig. 63 that at no time can an inferior planet and the sun be in opposite side of the earth. This is because its orbit is smaller than the orbit of the earth.

Conjunction : A body is in conjunction with the Sun, when both the Sun & the body are on the same side of the earth on the same meridian. In the case of an inferior planet, as in fig. 63, whether it is at V_1 or V_3 the sun & the body are on the same side of the earth. Hence both points are called conjunctions. For purpose of distinction position V_1 is called **inferior conjunction** & V_3 as **superior conjunction.**

Fig. 64 shows the orbit of a superior planet. In this case the position of the planet at J_1 alone will satisfy the conditions for a conjunction because only in this position, both sun and the body are on the same side of the earth & on the same meridian.

Opposition : A body is said to be in opposition with the sun, when the body and sun are situated on opposite sides of earth. As stated earlier such a situation can never occur for an inferior planet. For a superior planet, as in fig. 64, the position J_3, satisfies this condition, where the body and the sun are in opposite sides of the earth & on opposite meridians.

Maximum Elongation is the maximum separation in longitude between the sun and the body and will occur when the angle subtended at the body between the sun and the earth is 90°. At this point the line of sight from the earth to the body is tangential to the orbit of the body. (See positions V_2 & V_4 in fig. 63). For purposes of distinction and for reasons to be explained later in this chapter, position V_2 is named maximum Elongation (West) and position V_4 is named maximum Elongation (East). It may be pointed out, maximum elongation as aforesaid occurs only in the case of inferior bodies. In case of superior bodies, the difference in longitudes between the sun & the body can be as much as 180° apart, when they happen to be in opposition, and hence the terms maximum elongation is meaningless in their case.

Quadrature : In fig. 64, position J_2 & J_4 are the quadrature positions for the superior planet. In these positions, the angle formed at the earth between the sun & the planets will be 90° Such a situation will not arise for an inferior planet.

Sidereal Period of a body is the time taken by an orbiting body to complete 360° around its orbit relative to a fixed point in space.

Synodic Period of a body is the time taken by an orbiting body to go from one conjunction with the sun, to the next conjunction with the sun. Because of the earth's orbital motion, the synodic period of an inferior planet will invariably be longer than the planet's sidereal period. For a superior planet, its synodic period will be greater than the earth's sideral period. In fact the greater the distance of the superior planet from the sun closer will its synodic period be to the earth's sidereal period.

This is because, during one sidereal period of the earth, the arc by which a far off planet has moved on its orbit is fairly small which the earth traverses is comparatively shorter time to form conjunction with the body & sun again.

Venus as a morning & Evening Star

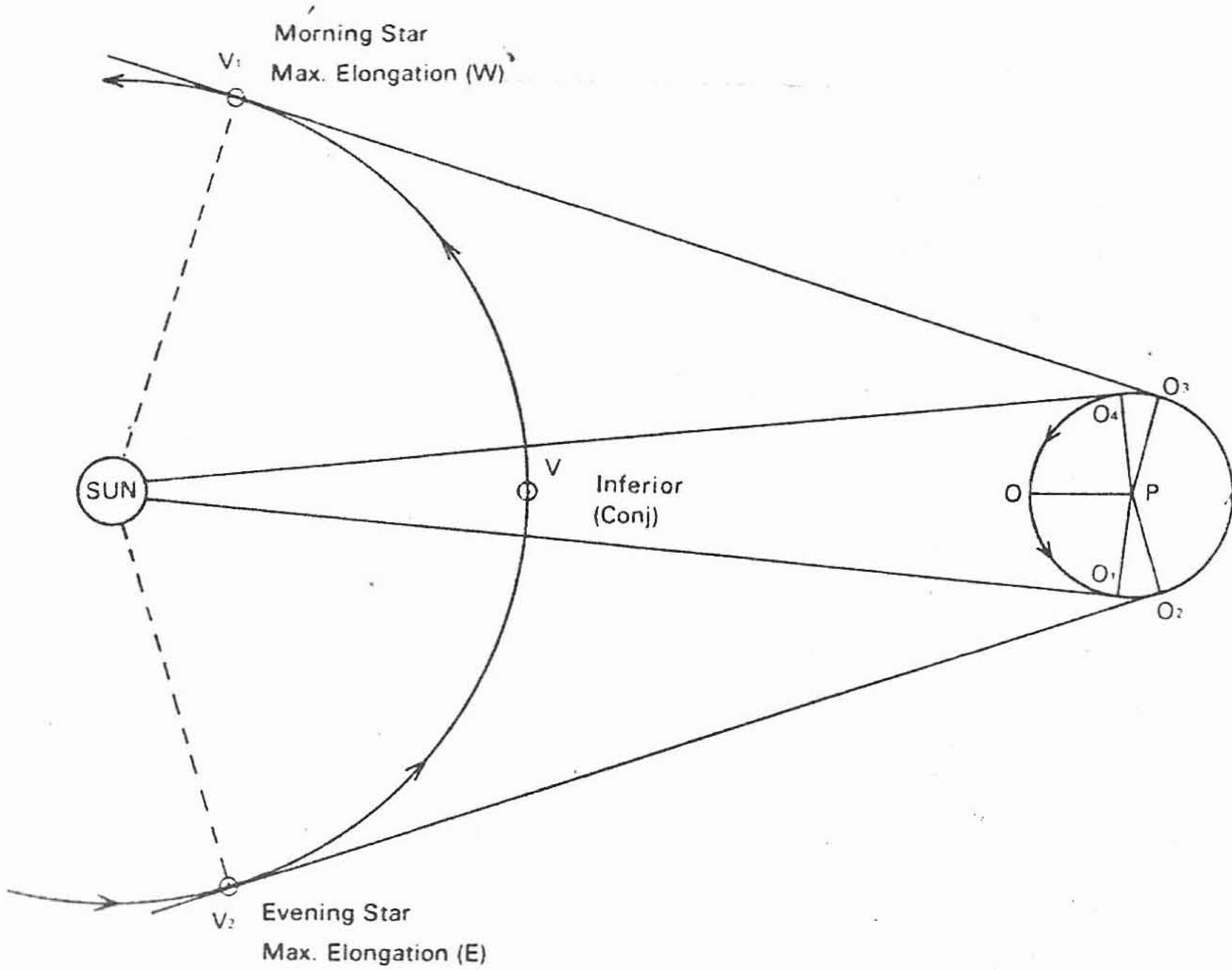


Fig. 65

Consider the Sun, Venus (V) and the Earth in the respective position as shown in fig. 65. The earth rotates anticlockwise on its axis as viewed from space on to the North pole. The planet Venus goes round its orbit around the sun in the direction of the arrow shown. When Venus is at V ie. at inferior conjunction, both Venus & Sun will rise, cross the meridian & set together for an observer (O) on the earth's surface. In figure, when observer reaches position O_4 both Sun and Venus will rise, cross the meridian when observer is rotated to O and set when he reaches O_1 For convenience let us keep the earth stationary and let Venus move on its orbit. Day after day, as Venus progresses in its orbit, the rays of Venus & Sun, become more and more divergent and when Venus reaches position V_1 the maximum separation between the rays takes place. In this position Venus will rise when observer has been rotated to a position O_3 whereas the sun will rise only when observer reaches O_4. Thus we see Venus rises before the sun and behaves like a **morning star**. In fact, Venus begins to be a morning star, no sooner it leaves the inferior conjunction position, and the interval between the Venus rise & Sun rise keeps on increasing till the maximum interval occurs at V_1. Since Venus rises before the Sun, it must be west of the sun as seen from the earth. The position V_1 is hence termed **maximum elongation (West).** As Venus proceeds on its orbit further, it will be observed that Venus'rays and Sun's rays are again closing on to each other, till when Venus reaches, superior conjunction, when again Sun and Venus will rise, cross the meridian & set together.

Subsequent positions of Venus on its orbit will again produce divergence of Sun's and Venus's rays, till a maximum separation occurs when Venus reaches position V_2. In this position, it will be observed from figure that the sun sets when observer reaches O_1 whereas Venus sets only when observer is rotated to O_2. Thus Venus is seen even well after Sunset. Hence Venus is now called an **evening star.** The position of Venus is now East of the Sun as seen from the earth. The position V_2 therefore is termed the **maximum elongation (East).**

As Venus proceeds on its orbit further, it will be observed that rays of Sun & Venus are again closing up till position V is reaches when they will again rise, cross the meridian and set together.

Thus we see that Venus behaves as morning star from inferior conjunction, through maximum elongation (West) to superior conjuction. From superior conjunction through maximum elongation (East) to inferior conjunction it behaves as an evening star.

Though for purposes of demonstration, we kept the earth stationary in fact the earth also has a motion on the orbit in the same direction as Venus. Because the rate of motion of Venus on its orbit is more than that of the earth, Venus will overtake the earth at inferior conjunction and the phenomenon explained above will take place, but stretched over a longer period than what may be evident from the figure 65. The whole cycle repeats again every Synodic period of Venus which is about 584 days.

As Venus closes on to inferior or superior conjunction, the rays will be so close to the sun that Venus cannot be visually observed from the earth due to sun's brightness. Its maximum brightness will occur near the two maximum elongations.

As Venus goes from inferior conjunction to inferior conjunction again, at various positions in its orbit, it presents phases similar to that of the moon. Though this is not directly evident to the naked eyes, observations through a powerful telescope will revel the phases clearly.

At maximum elongations, the difference in longitudes of Sun and Venus is about 47°. Hence in tropical latitudes, the earliest Venus can rise is about 3 a.m. local time and latest it can set is about 9 p.m. depending on whether the body is morning or evening star during that period. Therefore, Venus cannot be observed, close to midnight in tropical latitudes.

In high latitudes, depending on the declination of Venus, when it approaches circumpolar conditions, Venus can be observed close to midnight.

Similar phenomenon, can also occur in the case of Mercury, which is the other inferior planet. Since Mercury is much closer to the Sun than Venus, its maximum Elongations are about 20°. Hence earliest it can rise is about one hour before sun rise and latest it can set is about one hour after sun. Even at these times, since the body will be too close to the horizon, it will be difficult to observe it, either due to brightness of day light or cloudy skies, or morning or evening haze close to the horizon. All other times its rays will be too close to the sun for observation.

Decrease & Increase of S.H.A.of Planets.

Venus : In figure 66, assume earth to be stationary at position E and Venus moving on its own orbit from position V at inferior conjunction. In this position both Venus & Sun will be projected at X in the sky & their SHA will be indicated by arc ♈X, measured westward as shown by arrow in figure. When Venus moves to V_1, it will be projected at X_1 and its SHA will be arc ♈X_1 having registered an increase. At V_2, point of projection is at X_2 and

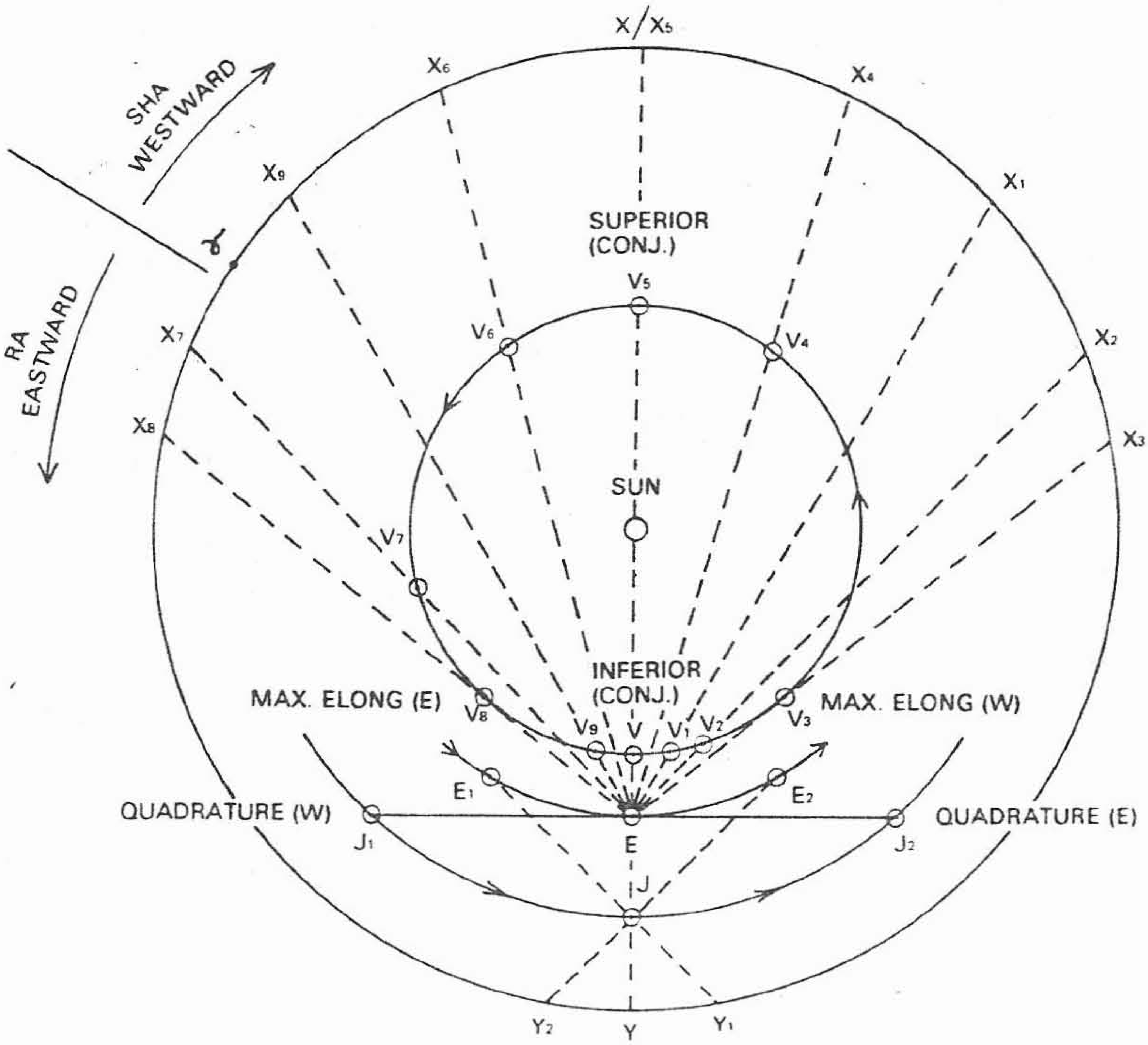


Fig. 66

Outer circle reps. Equinoctial.
♈ reps. Position of 1st point of Aries.
V V_1 V_2 etc. reps. Position of Venus on its orbit.
E E_1 E_2 etc. reps. Position of earth on its orbit.
J reps. Superior planet Juipter.
X X_1 X_2 etc. reps. Points of projection of Venus in sky.
Y Y_1 Y_2 etc. reps. Points of projection of Jupiter in sky.

corresponding SHA of Venus is arc ♈X_2 again showing an increase. At V_3 Venus is at its maximum elongation (West) and its SHA then is arc♈X_3. Any further advance of Venus on its orbit will place its point of projection closer & closer to ♈, such as at position V_4, the point of projection is X_4 & SHA is ♈X_4. This is less than ♈X_3 thus showing a reduction in SHA. At V_5 the body is at its superior conjunction, and the point of projection for Sun & Venus are at X_5. The SHA of both again is ♈X_5; again a reduction from the earlier SHA. Similarly at V_6 the SHA is reduced to ♈X_6 and at V_7, the SHA is the larger arc ♈X_7. Even though now, the point of projection is East of ♈ SHA being measured westward, has registered a decrease, since it is less than 360°. At V_8 Venus has reached its maximum elongation (East) and is projected at X_8 showing its least SHA as the larger arc ♈X_8. Any further movement of Venus on its orbit, such as at V_9, the point of projection falls at X_9, giving ♈X_9 as its SHA, which has now increased. Finally when Venus reaches V its SHA again increases to ♈X, same as that of the sun. We thus see that from maximum elongation (W) at (V_3) through superior conjunction (V_5) to maximum elongation (E) at (V_8) the SHA, decreases and from maximum elongation (E) at (V_8) to maximum elongation (W) at (V_3) through inferior conjunction (V) the SHA increases.

Since the sum of SHA & RA (Right Assension) is always 360° whenever there is a decrease in SHA, the RA increases and vice versa.

At V_1 Venus appears to be amongst a group of stars situated in the direction of X_1. Similarly at V_2 it is amongst a group of stars in the direction of X_2. To an observer on the earth, it would appear as if the planet Venus has moved from X_1 to X_2. This apparent backward motion of a planet on its orbit is called the **Retrograde motion.** An increase in SHA is always associated with the retrograde motion. When SHA is decreasing, the body appears to move in its correct direction in the orbit. This motion is called the **Direct motion.**

For purposes of demonstration we kept the earth stationary and moved only Venus on its orbit. In actual fact the earth is also moving on its own orbit, in the same direction as Venus. Since rate of orbital motion of Venus is greater than the earth, Venus will overtake the earth at inferior conjunction and the phenomenon explained above will take place. The period during which the increase & decrease of SHA taking place will therefore be stretched out to longer duration than what may be apparent from the figures.

Superior planets - (Jupiter)

The decrease & increase of SHA is not restricted to inferior planets alone. Even in the case of superior planets, Jupiter for example (See fig. 66) the SHA can increase at certain times.

Refering to fig. 66 this time let us move the earth on its orbit and keep Jupiter (J) stationary at J. At position J the Sun and Jupiter are in opposition. When earth was at E_1 the point of projection of J was at Y_1 and the SHA then was the reflex arc ♈Y_1 measured westwards. When earth comes to E, J is projected at Y & SHA then is the reflex arc ♈ Y which has shown an increase. Similarly when earth moves to E_2, Y_2 becomes the point of projection of J and the corresponding SHA is ♈Y_2 indicating a further increase.

In the case of superior bodies from its position a little after quadrature (West) to a position little before quadrature (East) the SHA will increase. At other times it will decrease.

Here again an increase of SHA is associated with the retrograde motion of the planet, because viewed against a back ground of fixed stars, the planet Jupiter appears to have moved westwards from Y_1 to Y_2

SHA of Sun always decreases (See Fig. 67)

Let Sun be at vernal equinox to start with. When earth is at E, the Sun (S) is projected at X, ie. at Vernal Equinox. When earth moves to E_1 the point of projection of Sun is at X_1 & SHA is the reflex arc ♈ X_1 measured westwards. At E_2, SHA of sun is ♈X_2 measured westwards. Similarly at E_3 it is ♈ X_3 and so on for various positions of the earth on its orbit. It is evident from the figure that the SHA is continuously decreasing.

'v' correction in the Nautical Almanac. The GHA of the Sun & the planets are computed for tabulation in the alamanac, assuming that they increase their HA at the rate of 15° per hour. Though this is true for the sun, it is not so for planets, because of their own orbital motion. The hourly increament in the GHA for

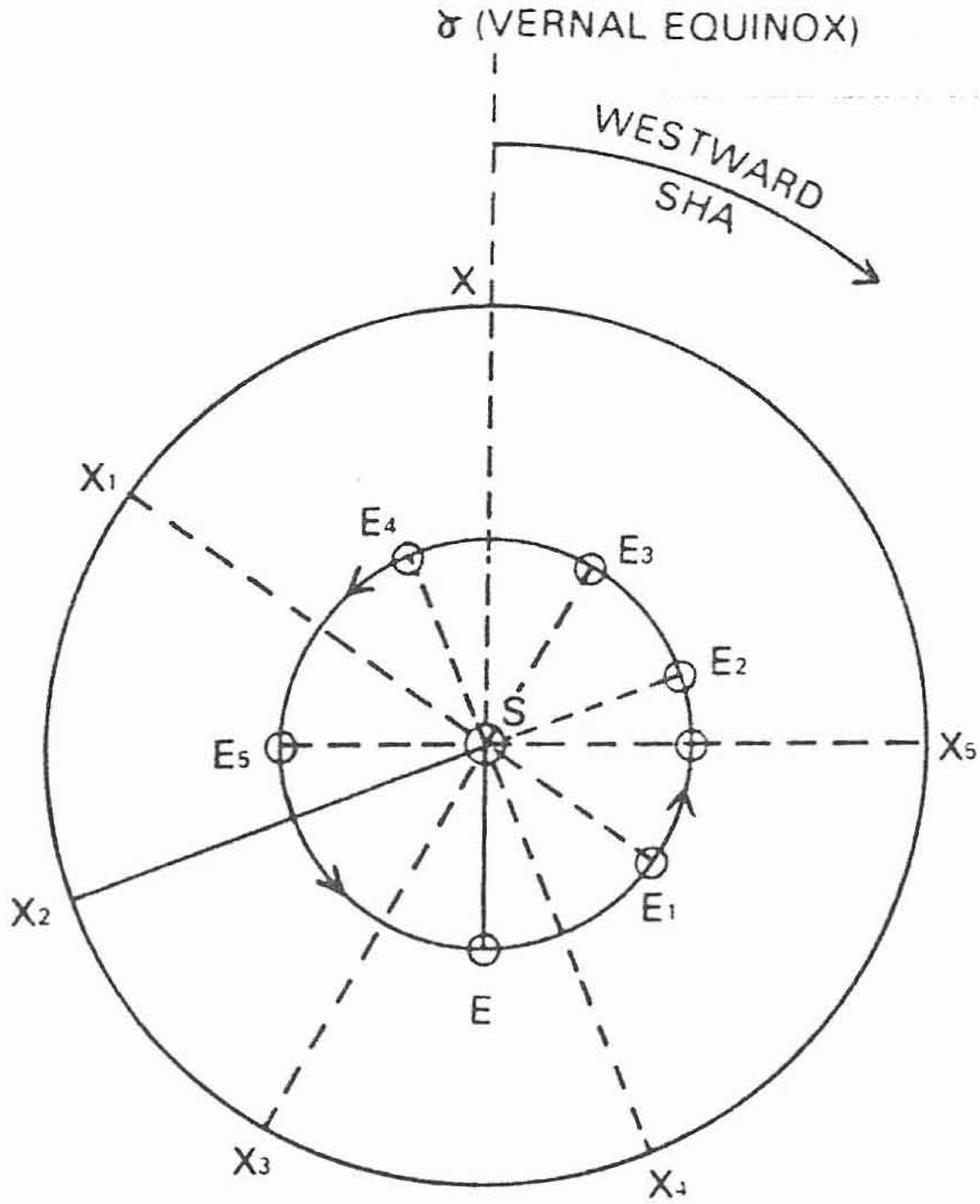


Fig. 67

planets over & above what is allowed for tabulation is termed 'v' correction for the planets & Moon. This value is always positive for all bodies except Venus, whose 'v', can be negative sometimes. This occurs only when Venus is moving eastwards relative to the Sun. At this time its SHA will decrease at a greater rate than the sun. In the case of all other bodies 'v' correction is positive.

In the case of the moon, the GHA tables are tabulated for a minimum increament of 14° 19' per hour. Any relative orbital hourly motion producing an excess over this amount is tabulated as 'v' correction for moon.

In the case of Aries the GHA tables are tabulated at the rate of increase of HA at 15° 02.46' per hour. Since the sun has to move from Aries to Aries ie. through 360° in one tropical year, the hourly decrease in SHA of sun $= \dfrac{360°}{\text{No of hours in a tropical year}} = 2.46'$.

This makes Aries gain on the Sun by 2.46' per hour which becomes a constant 'v' correction for Aries. This also accounts for the fact that compared with Sun, Aries comes on the meridian about 4 minutes earlier each day.

Exercise X

(1) Define the following terms :-
Inferior planet, superior planet, inferior conjunction, superior conjunction, sidereal period, synodic period, retrograde motion, maximum elongation.

(2) Why does Venus sometimes appear as a morning star and sometimes as an evening star ?

(3) Why is it not possible to see Venus, close to midnight in tropical latitudes ?

(4) Why does the SHA of Venus sometimes increase & sometimes decrease ?

(5) Why does the SHA of Sun always decrease ?

(6) On a certain day at 00 hrs. GMT. GHA of Moon was 114° 30.5' and at 01 hrs. GMT it was 129° 00.2'. What was the 'v' correction of moon for that hour ? (Ans. 10.7')

CHAPTER XI

TWILIGHT

Twilight is the term used to denote that period, before sunrise and after sunset, when the observer, on the earth's surface receives a diffused light. This is caused due to the following reasons. When the sun is just below the horizon before sunrise and after sunset, the sun's rays reach and illuminate the upper strata of the atmosphere. The atomsphere contains millions of suspended particles of dust, vapour, clouds etc. which scatter the rays of the sun. A substantial part of these scattered rays reach the surface of the earth directly below it and illuminates the area, which we term as twilight.

Theoritically, twilight lasts till the sun is 18° vertically below the horizon. For practical pruposes this arc of 18° is divided into three parts of six degrees each.

When the sun is between 0° & 6° below horizon, the period is called **Civil Twilight**, & from 6° to 12° below, it is termed **Nautical Twilight** and from 12° to 18° below it is called **Astronomical Twilight.** In practice it is difficult to specifically indicate as to when one type of twilight finishes and the next type starts. As one gradually merges with the other we can only vaguely differentiate one from the other.

For instance, astronomical twilight for all practical purposes is total darkness. Sea horizon cannot be clearly distinguished, and most stars except perhaps 5th & 6th magnitude stars are visible. To see anything on deck,artificial light will be needed.

During nautical twilight, the horizon can be clearly seen, and all important stars useful for navigational purpose are still visible, though majority of dim stars have gone out. This period therefore is the most suitable time to take star sights and hence called the nautical twilight. Silhouette on deck or outlines of distant ships can be seen, even though details cannot be made out.

Civil twilight, for all practical purposes, is day light. Most objects on deck can be clearly seen without any artificial light, the horizon is very clear, and most stars have gone out except perhaps the very bright ones. This period therefore is not suitable for star sights.

Civil twilight ends with sunrise in the morning and begins at sunset again. The nautical almanac gives the LMT. of sunset or sunrise, Civil Twilight and Nautical Twilight for various latitudes. Astronomical twilight is not tabulated therein because this has little practical use.

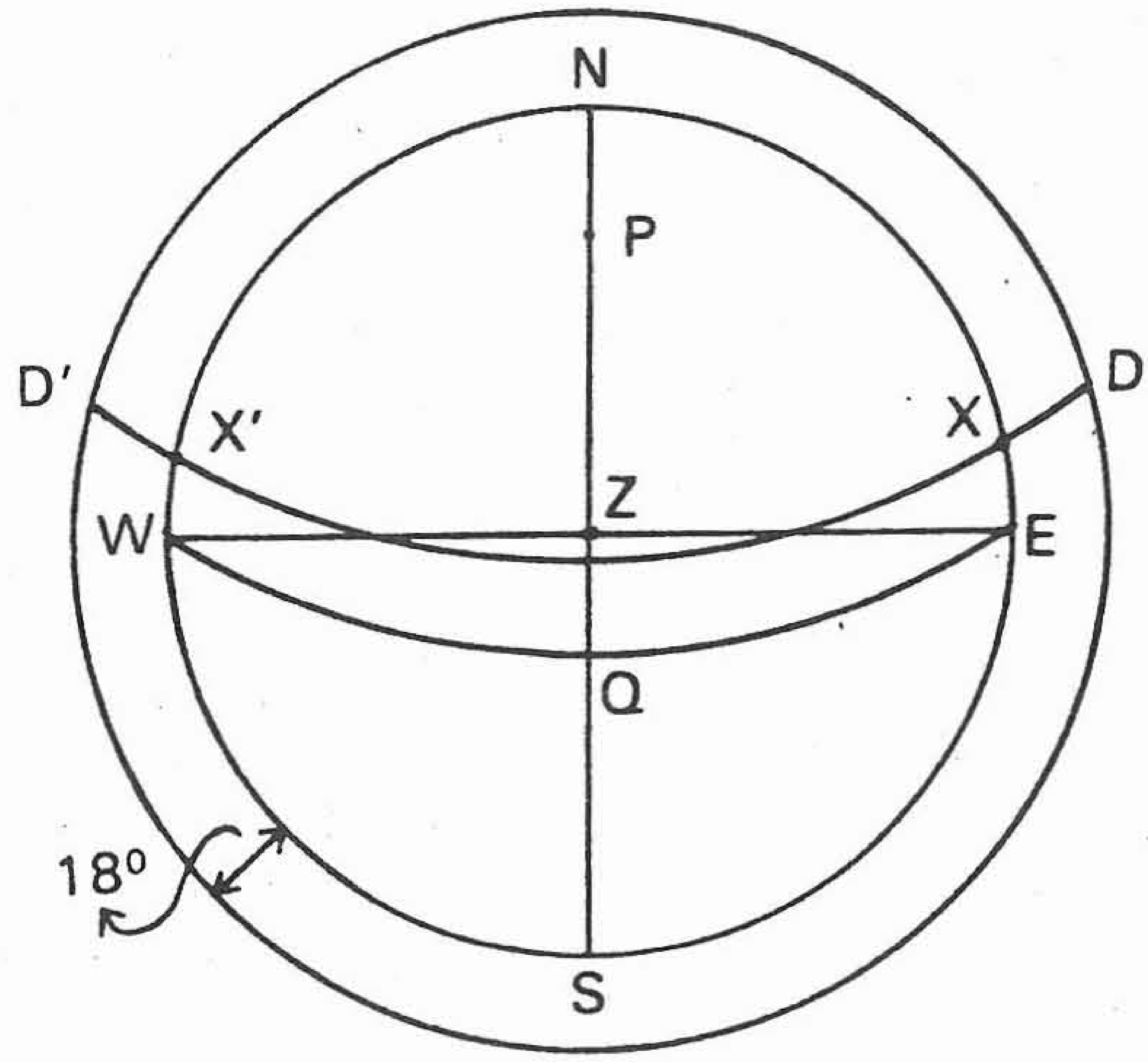


Fig. 68

WQE is the Equinoctial.
QZ = NP = Lat of observer.
D X X'D' = Parallel of Declination of Sun.

The outer circle represents an arc 18° directly below the horizon, which is shown as if pulled up above horizon for convenience of demonstration.

Twilight starts in the morning from the time the Sun reaches position 'D' and lasts till it rises at X and similarly, in the evening the twilight starts from the time the sun sets at X' and lasts until it reaches D' It should be noted that the latitude (QZ) being small the sun traverses the 18° twilight arc, more vertically from D to X and hence twilight lasts for a lesser duration in lower latitudes as will be seen in comparison with fig. 69 which is drawn for a higher latitude.

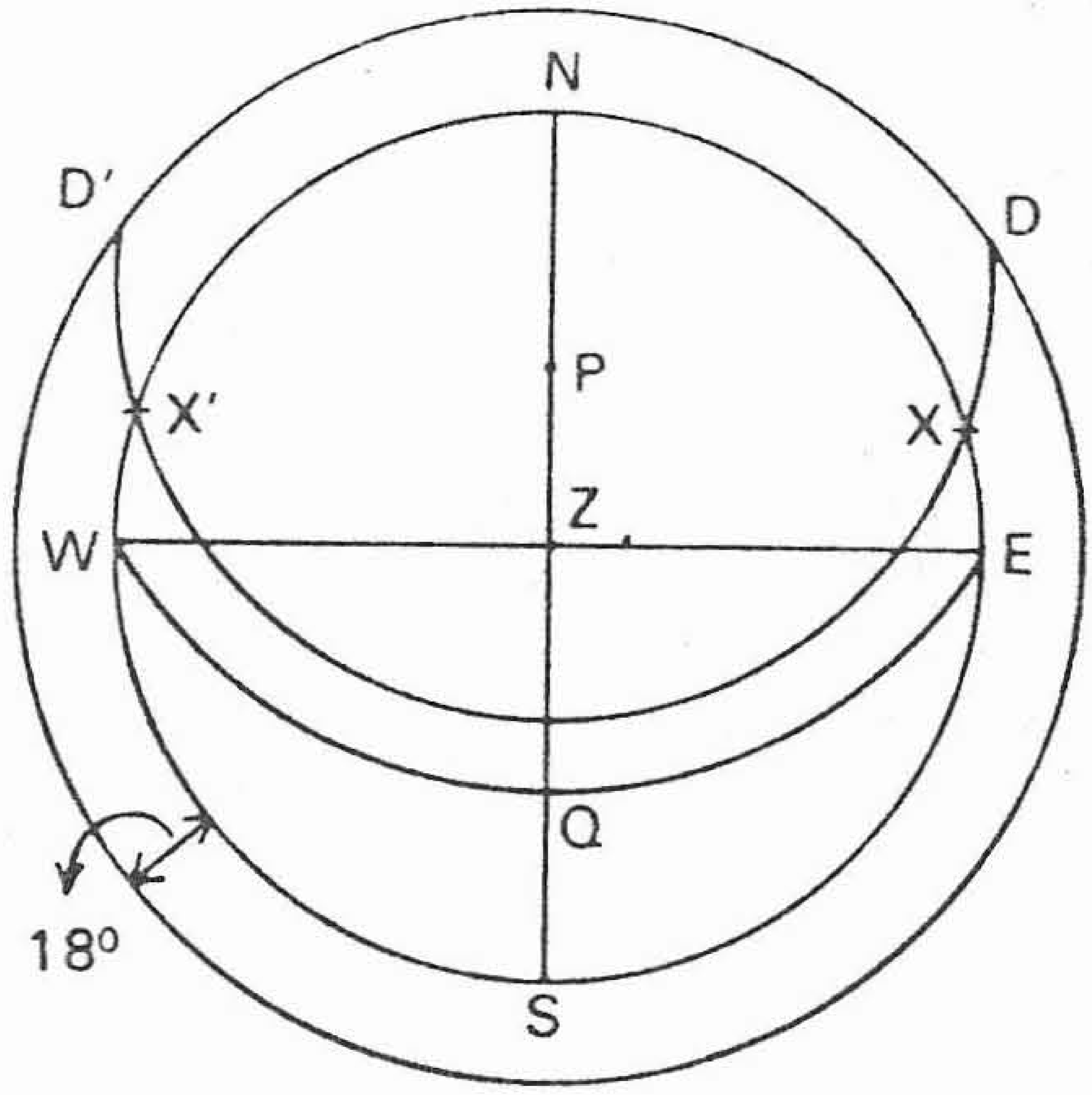


Fig. 69

Given the same declination of the sun, if the latitude (QZ) is increased to a larger value as seen in fig. 69, the arc DX or X'D' is covered by the sun more obliquely thus taking longer time to cross the 18° vertical arc below the horizon. Hence, the twilight lasts longer in higher latitudes than in lower latitudes.

If one keeps on increasing his latitude, on the same day ie. for the same declination of the sun, a stage will be reached when

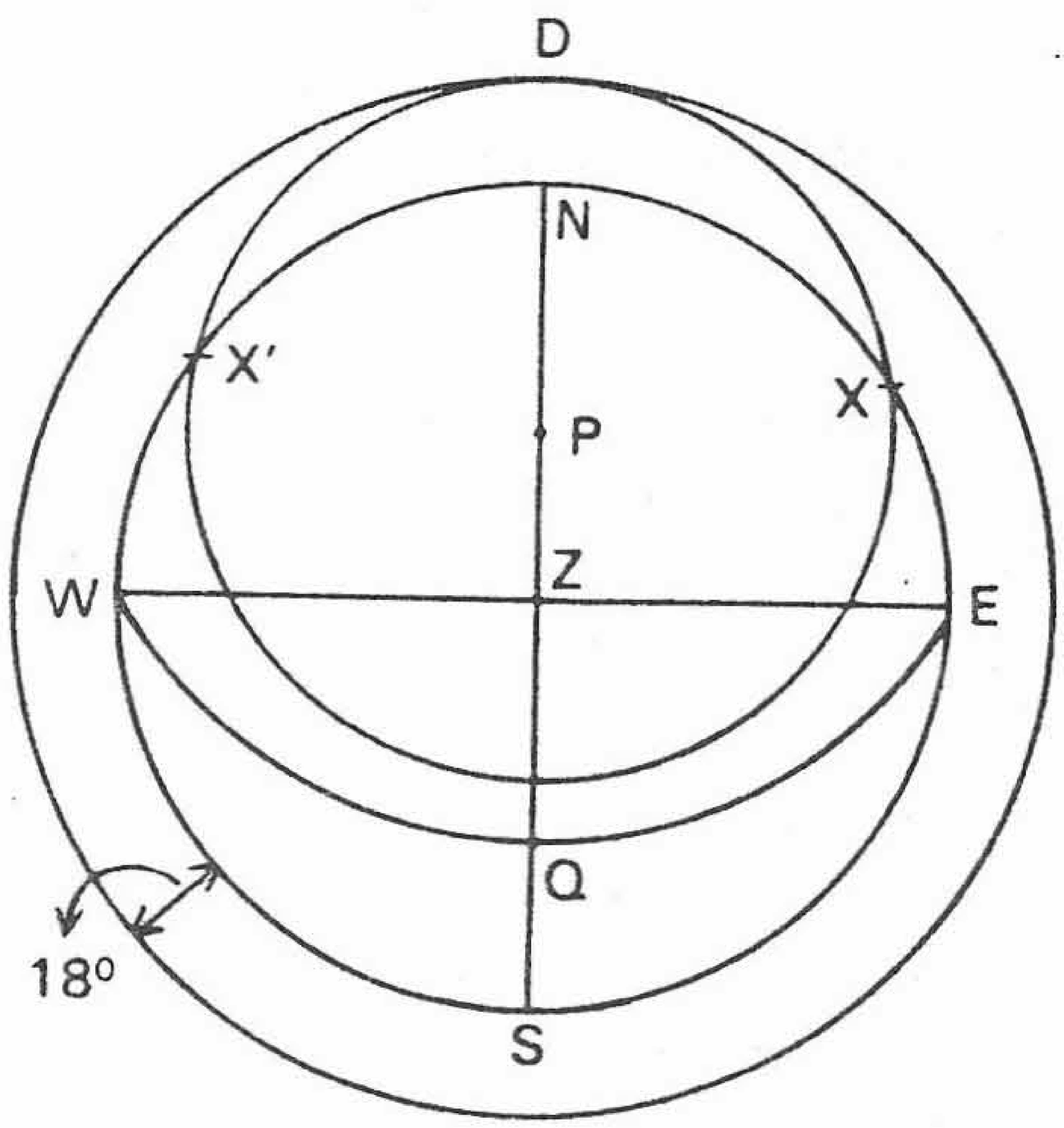


Fig. 70

the parallel of declination itself will not dip below the 18° vertical arc below the horizon. This is illustrated in fig. 70.

It will be observed from the fig. 70 that the parallel of declination DX' remains wholely within the 18° arc below the horizon. Though the sun will rise at X and set at X', there will not be total darkness anytime during the night, because the sun does not go below the 18° arc vertically below the horizon. Such latitudes will experience **Twilight all night** (TAN).

The lowest Lat. where TAN will occur, will be where the declination circle just skims the 18° arc below the horizon as seen in fig. 70 where :

NP = Lat. = QZ
ND = 18° Arc
PD = Polar distance of sun = (90–Decl.)
Lowest Lat. where TAN will occur is:
Lowest Lat = Pol. Dist. – 18°

The same phenomenon will continue to occur till the latitude is increased by another 18° ie. when point D, coincides with N. Such a latitude is the highest latitude where TAN will occur.

At that time PD = PN

∴ Highest lat for TAN = Lowest lat + 18°
ie. Highest lat for TAN = Polar dist. of sun.

Since we used 18° as the twilight arc, the twilight discussed so far is for those latitudes where there will not be total darkness during the night.

A similar argument could be used for latitudes, where there will not be astronomical or nautical twilight. Latitudes where the sun will not go more than 12° below the horizon will not experience astronomical twilight. Likewise those latitudes where the sun will not go beyond 6° below the horizon will not experience astronomical or nautical twilight.

Conditions necessary for TAN phenomenon to occur are :–

(1) Latitude of observer & declination of Sun must be of same name.

(2) Latitude of observer must lie between P. D. of Sun and P. D.-twilight arc.

Worked Examples

(1) Given the decl. of Sun is 22° N, between what latitudes will there not be total darkness all night ie. will have TAN.

Lowest Lat = (90–22) – 18

= 68 – 18 = 50° N

Highest lat = 50° + 18° = 68° N

Between 50°N & 68°N there will be TAN.

(2) Between what lats will there be civil twilight all night when Sun's decl is 22°N ?

Lowest Lat = (90–22) - 6° = 68° - 6°= 62°N

Highest lat = 62°+ 6° = 68°N.

Between 62°N & 68°N there will be civil twilight all night

(3) Between what latitudes will there not be astronomical twilight when the sun's decl is 22°S ?

Lowest Lat for nautical TAN = (90 - 22) - 12

= 68 - 12 = 56°S

Highest Lat for TAN = (90 - 22) = 68°S

Between 56°S & 68°S there will not be astronomical twilight, but there will be nautical and civil twilights.

Exercise - XI

(1) What is twilight and how is it caused ?

(2) Distinguish between Astronomical, Nautical & Civil Twilights

(3) Why does twilight last longer in higher latitudes than in lower latitudes ? What conditions are necessary to have TAN ?

(4) On winter solstice day (Decl 23° 27′ S), find between what latitudes will there not be :

(a) Astronomical twilight.

(b) Nautical twilight.

Answers :

Q.4. (a) Between 54° 33′S & 66° 33′S.

(b) Between 60° 33′S & 66° 33′S.

CIRCUMPOLAR BODIES

A body is said to be circumpolar, when it remains above the horizon all day and therefore, does not arise or set. Such bodies will have two meridian passages, one called the upper meridian passage when the azimuth will change from east to west, the other called the lower meridian passage or crossing of the meridian below the pole when the azimuth will change from west to east. The meridian passage which is most often observed day to day is the upper meridian passage of a body.

In the last chapter we observed that the highest Lat for TAN is when the Lat of observer equals the polar dist. of the sun. If the latitude is increased any further, right upto 90° Lat, the sun will be circumpolar in all those latitudes. Though TAN is a phenomenon attributable only to the sun; circumpolar situations can happen to all bodies provided latitude & declination are suitable.

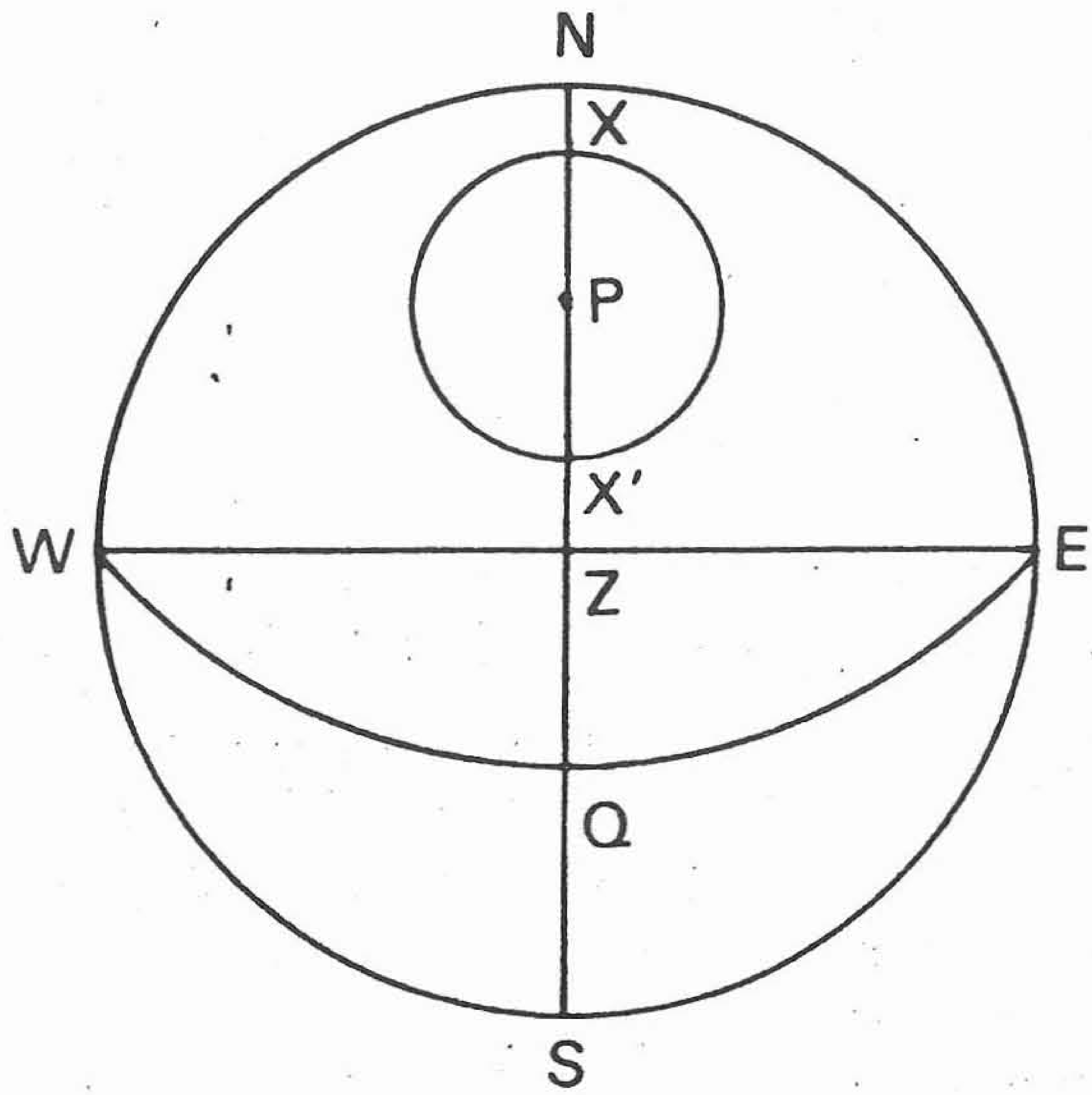


Fig. 71

NESW – Reps Obs rational horizon.
NZS – Reps Obs Meridian.
WZE – Reps Prime Vertical.

WQE - Reps Equinoctial.
P - Reps Elevated Pole.
NP = QZ = Latitude.
PX = PX' = Polar dist. of body.
X reps Pos. of body at Lower transit.
X' reps Pos. of body of Upper transit.
NX reps True alt. below pole.
NX^1 •reps True alt. above pole.

From the figure it will be seen that the conditions necessary for a body to be circumpolar are :

(1) Lat. of observer and Decl. of the body must have same names.

(2) Lat. $\geqslant$ Polar distance of the body

To find Latitude.

If the true altitude of a circumpolar body is known both at its upper and lower transits, the lat. of the observer can be calculated even without knowing what body it is. This is shown below:

$$\text{Lat. of the obs} = \frac{\text{Tr.alt.above Pole} + \text{Tr.alt.below pole.}}{2}$$

Proof : Refer to fig. 71 :-

$$NP = \frac{NX_1 + NX}{2}$$

$$= \frac{(NP + PX_1) + (NP - PX)}{2}$$

$$= \frac{NP + PX_1 + NP - PX}{2}$$

$$NP = \frac{2\,NP}{2} = NP \qquad \text{(Proved)}$$

Name the Latitude *same name as Azimuth of body at its Lower Transit.*

Occasions arise, when, the body bears south at upper transit and North at Lower transit or vice versa. In such a case, subtract the true altitude above pole from 180° and make it as if the altitude is taken from the same horizon as lower transit and then apply the formula given above. Name the latitude as per rule stated above.

To find Declination of the body

Declination of an unidentified body can also be calculated if the true altitude at upper & lower transits are known. This is as follows :

$$\text{Polar Distance} = \frac{\text{True alt. above Pole} + \text{True al. below pole}}{2}$$

Proof :- Refer to Fig. 71

$$PX = \frac{NX_1 - NX}{2}$$

$$= \frac{(NP + PX_1) - (NP - PX)}{2}$$

$$= \frac{NP + PX_1 - NP + PX}{2}$$

$$PX = \frac{2PX}{2} = PX$$

Declination = (90–PX) (Proved)

Name *Declination same name as Azimuth of the body at its Lower Transit.*

If bearing of body has different names at its lower and upper transits as said earlier, convert the altitude at upper transit by subtracting from 180° to an altitude as if it is measured from the horizon on the lower transit side, then apply the formula.

Declination can also be found after calculating latitude. Referring to fig. 71 :

NP – NX = PX 90 – PX = Decl.

Name declination same as latitude.

Method described so far assumes that there is no change of declination of the body between its upper & lower transit times. In the case of all stars and certain distant planets, this is true. But, in the case of Sun, Moon & near by planets, since there will be a change in declination between the time of upper and lower transits the method described cannot readily be applied to such bodies.

This method is therefore, ideally suitable for stars, provided they are suitable for observation at both transits. Both transits will differ in time by half a sidereal day viz. 11h 58m 02s.

Exercise XII

(1) What conditions are necessary for a body to be circumpolar ?

(2) Prove that $\frac{\text{Tr. alt. above pole} + \text{Tr. alt. below pole}}{2}$ = Lat. of obs.

How will you name the Lat. and why ?

(3) A star was, observed to have a true altitude on meridian above the pole 81° 04′ bearing S ~~[illegible]~~ and it had true altitude on meridian below the pole 16° 00′ bearing south. Find (a) latitude of the observer (b) declination of the body. (Ans. Lat 48° 32′S Decl. 57° 28′S).

CHAPTER XIII

SOLUTION OF SPHERICAL TRIANGLES

On a sphere a spherical triangle is formed by the intersection of the arcs of three Great Circles. All spherical triangles have six parts viz. three angles and three sides. Whether it be an angle or a side, they are all expressed in degrees, minutes & seconds of arc.

The properties of a spherical triangles are :

(1) The sum of the three angles will be anywhere between two and six right angles.

(2) The sum of the three sides together is less than four right angles.

(3) As in a plane triangle, the greater angle has the greater side opposite to it.

(4) When two great circles intersect, the vertically opposite angles are equal.

A spherical triangle can be :

(a) A right angled triangle, where one angle is 90°

(b) A quadrantal triangle, where one side is 90°

(c) An oblique spherical triangle where none of the angles or sides is a right angle.

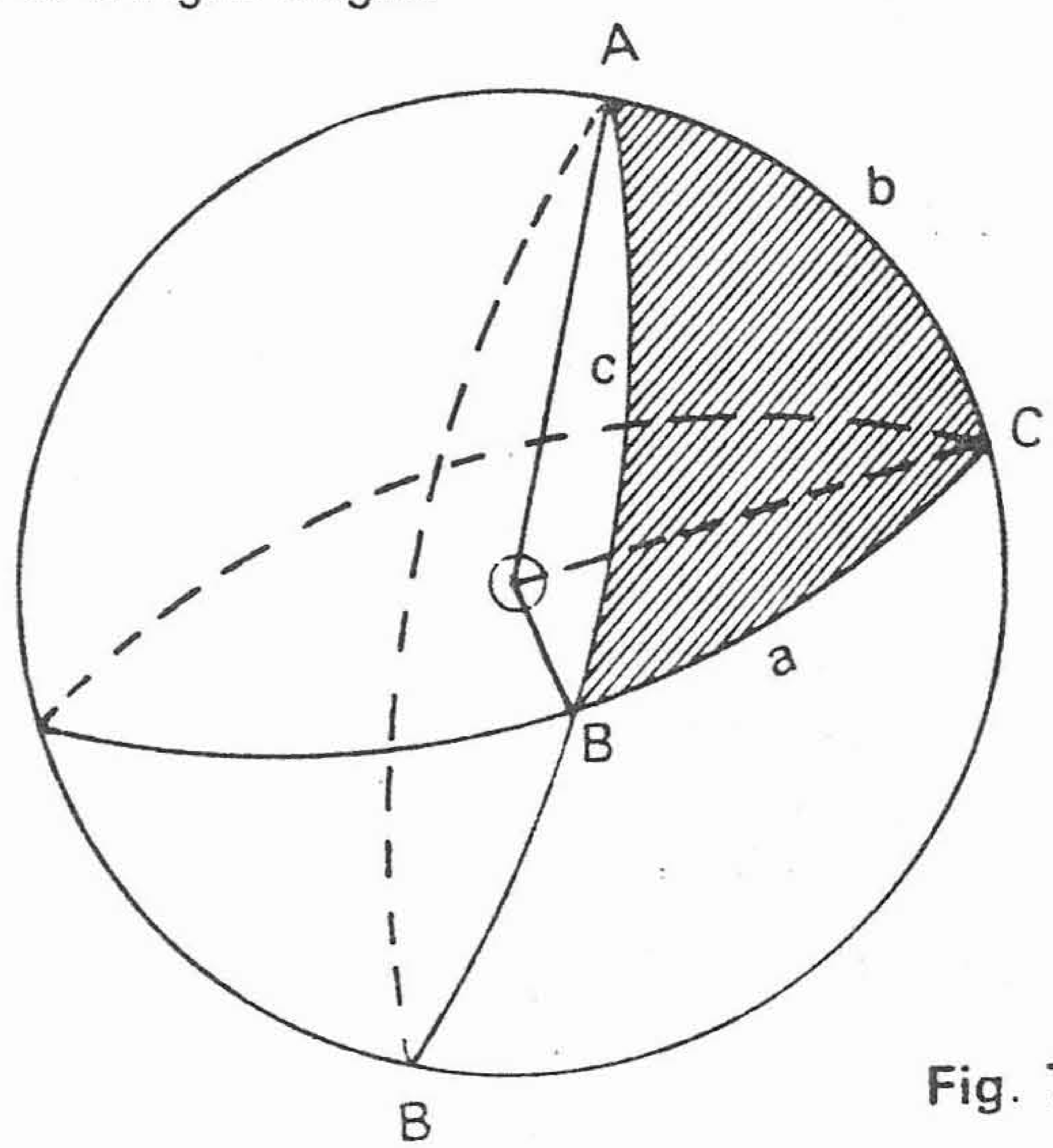


Fig. 72

In fig. 72 $\triangle$ ABC is an oblique spherical trangle.
AB, BC, AC = Arcs of three G.Cs.
It has 3 angles, A, B, & C & 3 sides a, b, and c.
O reps the centre of the sphere.

Right through practical navigation, we continuously apply the principles of solution of one or the other type of triangles.

I Solution of Right Angled Spherical Triangles

In many calculations in principles of navigation we may have to solve a right angled or a quadrantal spherical traingle.

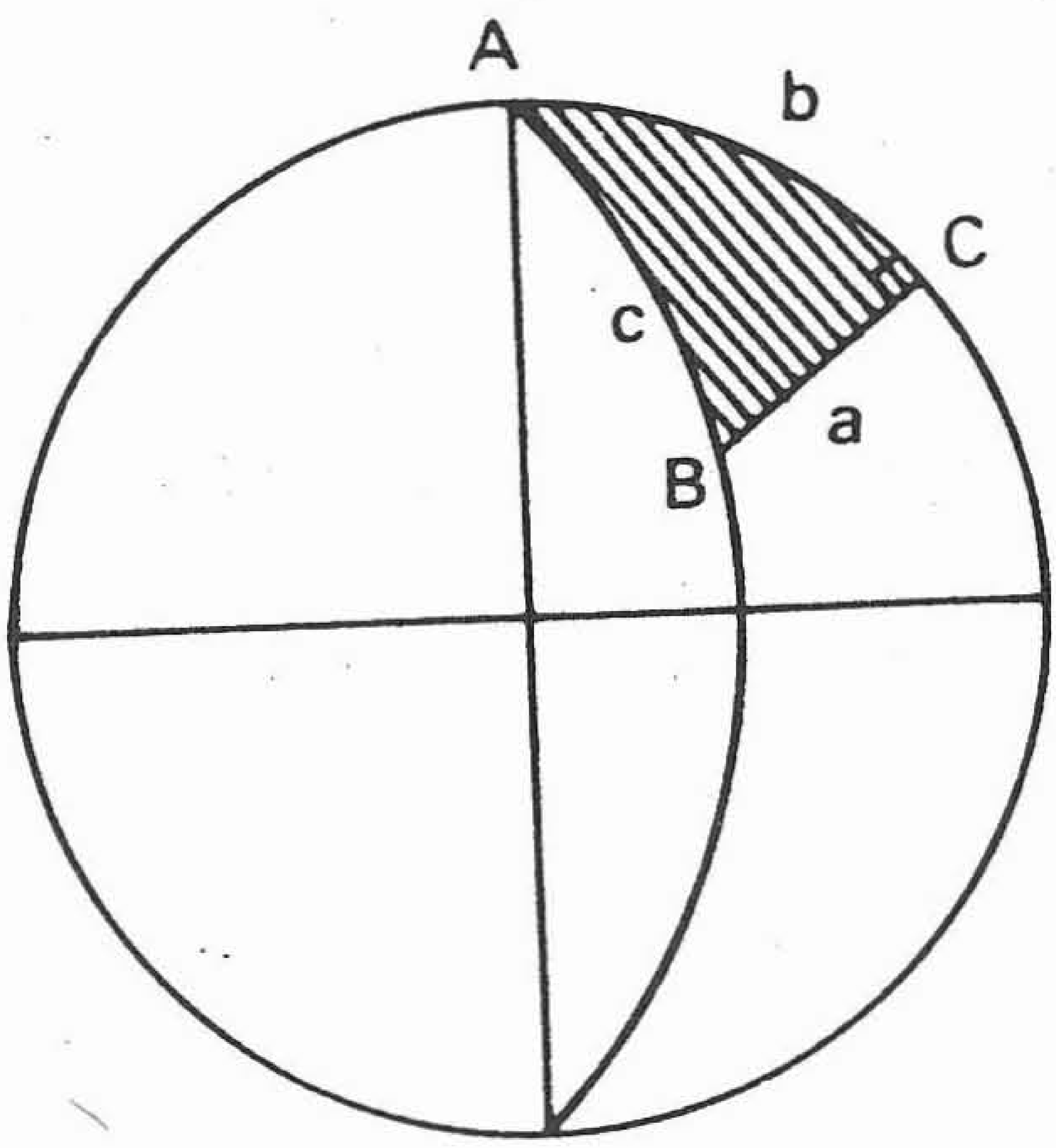


Fig. 73 (a)

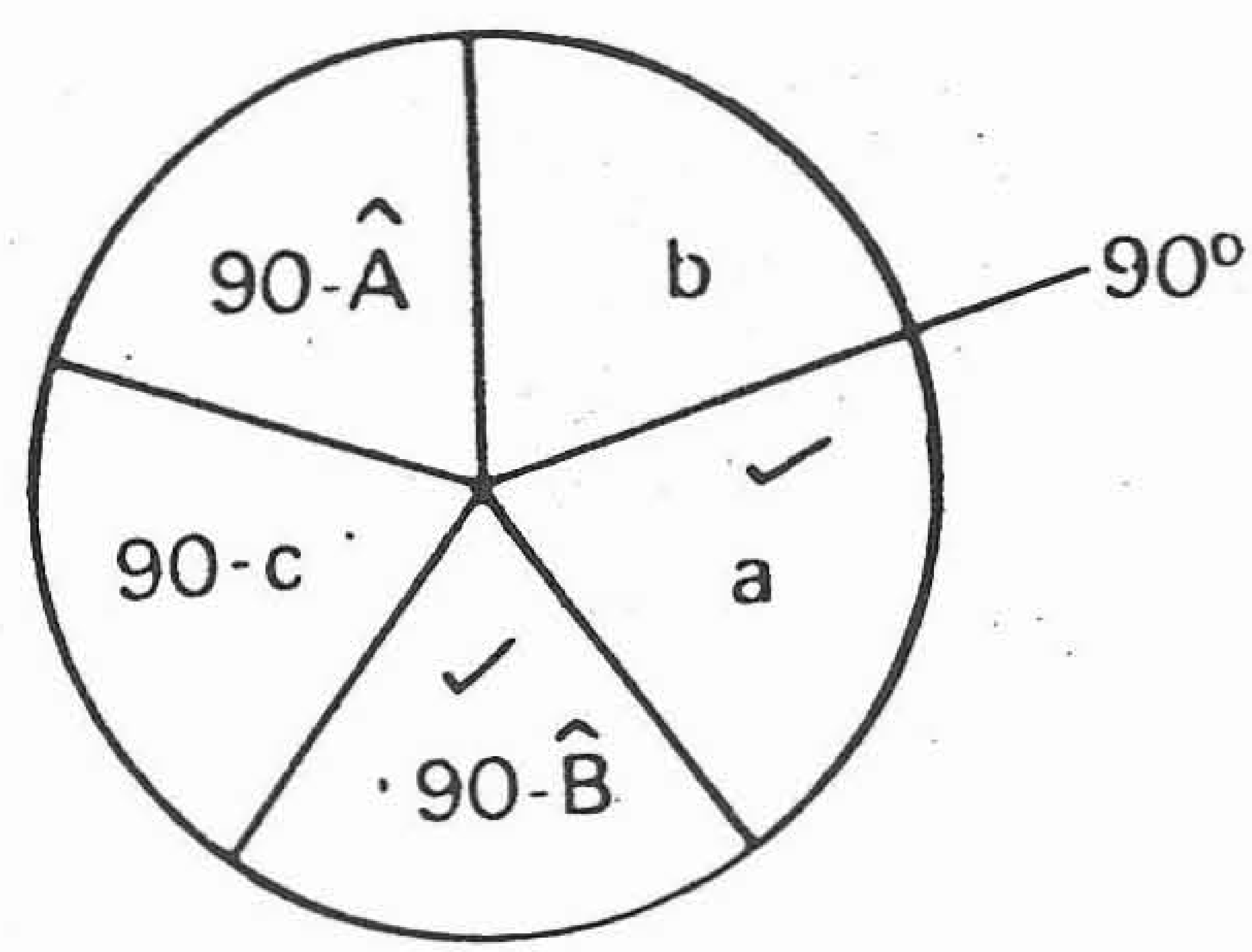


Fig. 73 (b)

Consider a right angled triangle ABC as in fig. 73 (a)
It has three sides a, b, c & two angles A & B
Excluding the right angle at C, it has only 5 parts.

Write down these five parts as in fig. 73 (b), in the same order as they appear in the triangle, but omitting the right angle. The sides adjacent to the right angle are written as they are, and the remaining 3 parts are written as (90 - X) ie. complement of respective side or angle.

Of the five parts, if any two parts are given, then the remaining three parts can be found by using the:

"Napier's Rules for Circular Parts", which states :

(1) Sine of the middle part = Product of **TANGENTS** of **ADJACENT** parts.

(2) Sine of the middle part = Product of **COSINES** of **OPPOSITE** parts.

Consider fig. 73 (a)

Suppose, you are given in the question values of side 'a' and $\hat{B}$ and that $\hat{C} = 90°$. You are required to find $\hat{A}$, side c & side b. We place these five parts as in fig. 73 (b). Mark off 'a' & $\hat{B}$ with a tick, to indicate we know these values.

The term **adjacent** in the Rule means touching each other eg. b and (90 - B) are adjacent parts for 'a', similarly 'a' & (90 - A) are adjacent parts for 'b'.

The term **opposite** means, parts which are **not** touching each other eg. for 'a', (90 - A) & (90 - c) are opposites - similarly for 'b', (90 - c) & (90 - B) are opposite parts & so on.

Since we know a & $\hat{B}$, we proceed as follows :

(i) **to find 'b'**

Sin a	=	tan b x tan (90 - B)	(Rule 1)
tan b	=	sin a x cot (90 - B)	
tan b	=	sin a x tan B	

By use of logarithams we can now find the value of 'b'.

To find 'c'

Sin (90 - B)	=	tan a x tan (90 - c)	(Rule 1)
tan (90 - c)	=	sin (90 - B) Cot a	
Cot c	=	Cos B Cot a	

By using Logs. value of 'c' can be obtained.

To find $\hat{A}$

Sin (90 - A)	=	Cos a x Cos (90 - B)	(Rule 2)
Cos A	=	Cos a x Sin B	

Use logs. now & find $\hat{A}$

The three unknown parts have now been found. Difficulties sometimes arise in deciding whether the angles and sides we just worked out are more than 90° or less than 90°. To clear that doubt **Some Axioms** have to be remembered.

When Sides or Angles are all over or under 90° they are said to be of **"Like Affection"**. When some are under 90° and others over 90° they are said to be of **"Unlike Affection"**.

The Axioms are :

(1) An angle and a side opposite to it are always of same affection ie. both over 90° or under 90°

(2) When two sides are of same affection, the third side is less than 90°; but when two sides are of unlike affection, the third side is more than 90°

(3) When the included angle between the hypotenuse and a side is under 90° then the two sides are of same affection ie. either both over 90° or both under 90°. But if the included angle is more than 90°, then both sides are of unlike affection

Whenever you decide that the angle is more than 90°, subtract calculated angle from 180° and the resulting value will be the answer.

In practice however it is not necessary to remember all these rules, because, the signs of trigonometrical ratios of angles, exceeding 90° will automatically adjust themselves and a final positive or negative value will automatically indicate which quadrant the angle is.

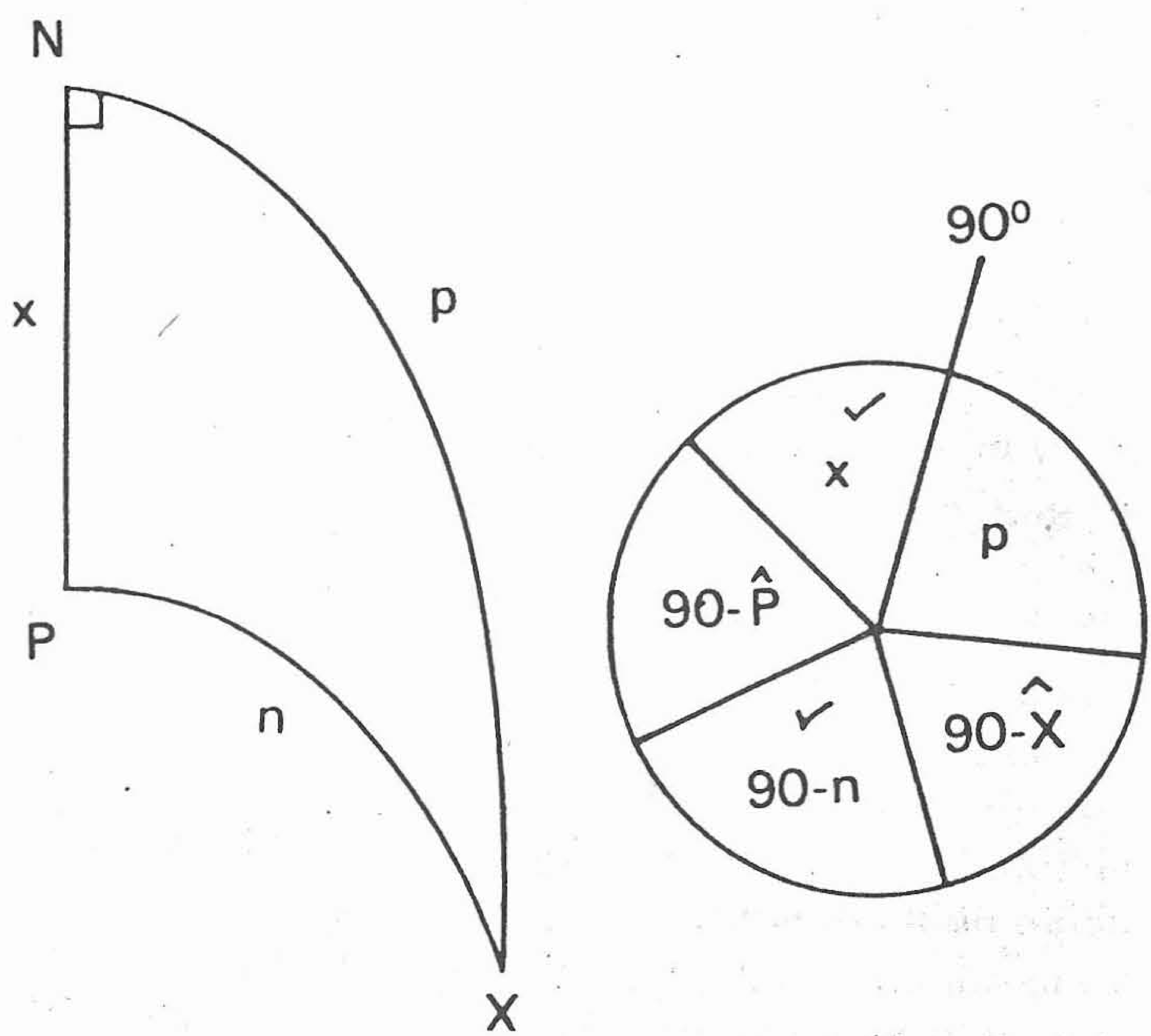


Fig. 74

Worked Example : In $\triangle NPX$ $\hat{N} = 90°$, side NP = 34° 10′ side PX = 112° 15′, find all the remaining parts. (see fig. 74)

Given NP = x = 34° 10′

PX = n = 112° 15′

$\hat{N}$ = 90°

To find NX, $\hat{P}$ & $\hat{X}$

(1) to find '*p*'

$\sin(90 - n) = \cos x \cos p$

$\cos p = \cos n \sec x$

$\cos p = \cos 112°\,15' \times \sec 34°\,10'$

No.	Rat.	Log.	
112° 15′	Cos	9.57824	(– ve)
34° 10′	Sec	0.08228	(+ ve)
62° 46′	Cos	9.66052	(– ve)

Since final value of Cos is –ve; the side lies in the 2nd quadrant.

$\therefore p = (180 - 62°\,46') = 117°\,14' = NX$

(2) To find $\hat{X}$

$\sin x = \cos(90 - X) \cos(90 - n)$

$\sin x = \sin X \;\; \sin n$

$\sin \hat{X} = \sin x \;\; \operatorname{cosec} n$

No.	Rat.	Log.	
34° 10′	Sin	9.74943	(+ ve)
112° 15′	Cosec	0.03361	(+ ve)
37° 21½	Sin	9.78304	(+ ve)

Since final answer is + ve, value lies in 1st quadrant. Further, because side x is less than 90°, $\hat{X}$ has to be less than 90°

$\therefore X = 37°\,21½$

To find $\hat{P}$

$\sin(90 - \hat{P}) = \tan x \;\; \tan(90 - n)$

$\cos P = \tan x \cot n$

No.	Rat.	Log.	
34° 10′	tan	9.83171	(+ ve)
112° 15′	Cot	9.61184	(– ve)
73° 52½	Cos	9.44355	(– ve)

Since final Cos ratio is - ve the angle falls in 2nd quadrant.

$\therefore P = (180 - 73^\circ\ 52\frac{1}{2}) = 106^\circ\ 07\frac{1}{2}'$

Ans : $NX = 117^\circ\ 14'$

$\hat{X} = 37^\circ\ 21\frac{1}{2}'$

$\hat{P} = 106^\circ\ 07\frac{1}{2}'$

Ambiguous cases :

If a side and an angle opposite to it are given, there will be two values for each of the remaining three parts. The two values for each part will be supplementary to each other. This is illustrated in fig. 75.

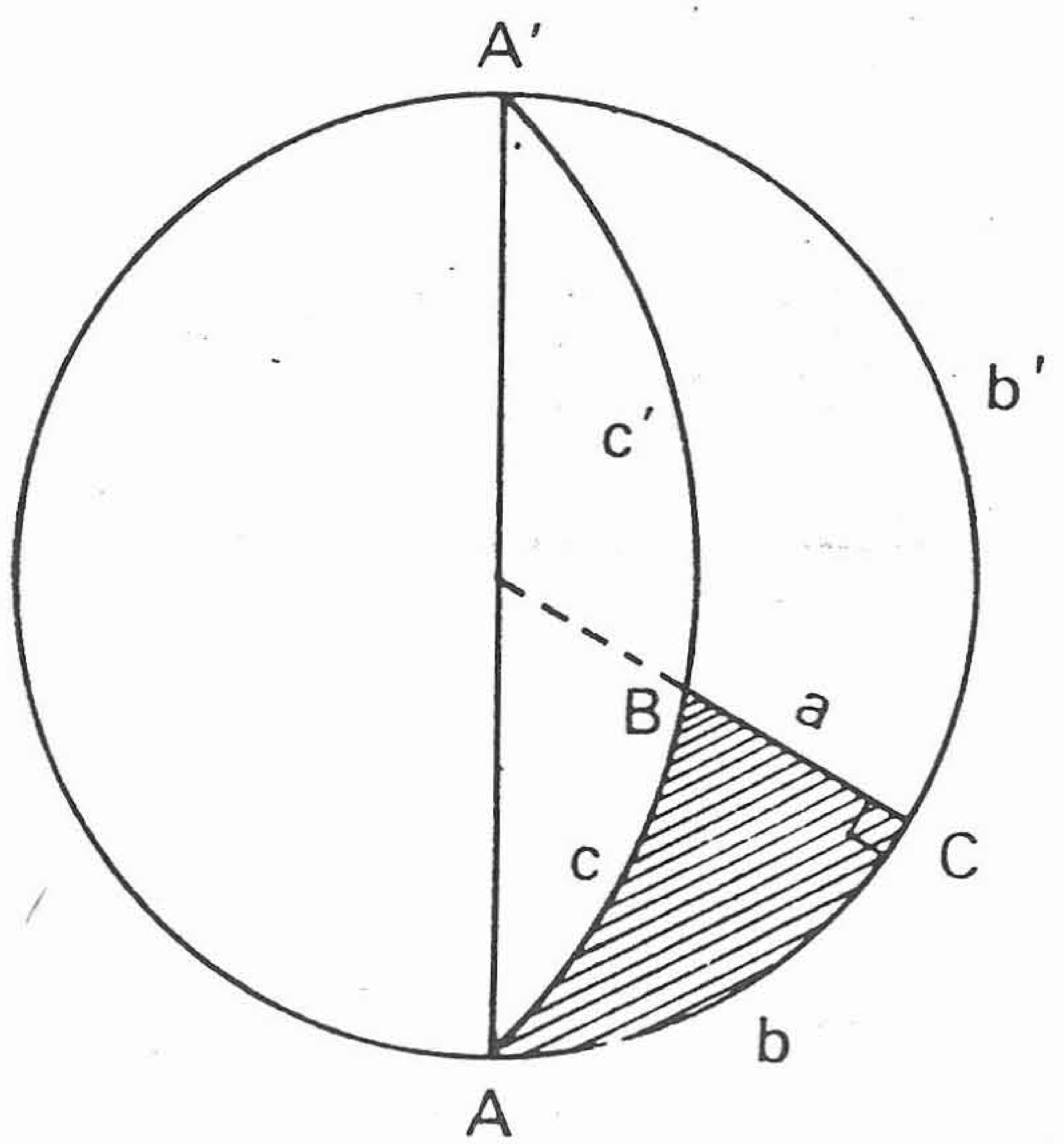


Fig. 75

In the right angled △ ABC

Given $\hat{A}$; side a & $\hat{C} = 90^\circ$

To find side b, side c & $\hat{B}$

It will be seen from the figure that even for △ A' B C, $\hat{A}' = \hat{A}$, side 'a' is common to both triangles. Hence side 'b' or b' and $A\hat{B}C$ or $A'\hat{B}C$ and side c or c' will satisfy the given data in the question.

Hence, ambiguous cases, such as these will produce two answers for each of the parts to be worked out.

II Solution of Quadrantal Triangles

Quadrantal trangles are those in which one side is 90°. These triangles can also be solved by using Napiers Rule for circular parts viz.

Sine of middle part = (1) Products of **Tangents** of **Adjacent** parts
or (2) Products of **Cosine** of **Opposite** parts.

The five parts omitting the quadrantal side are to be placed in the circle in the same order as they appear in the triangle, the two angles at the end of the quadrantal side as they are and complements of the remaining three parts.

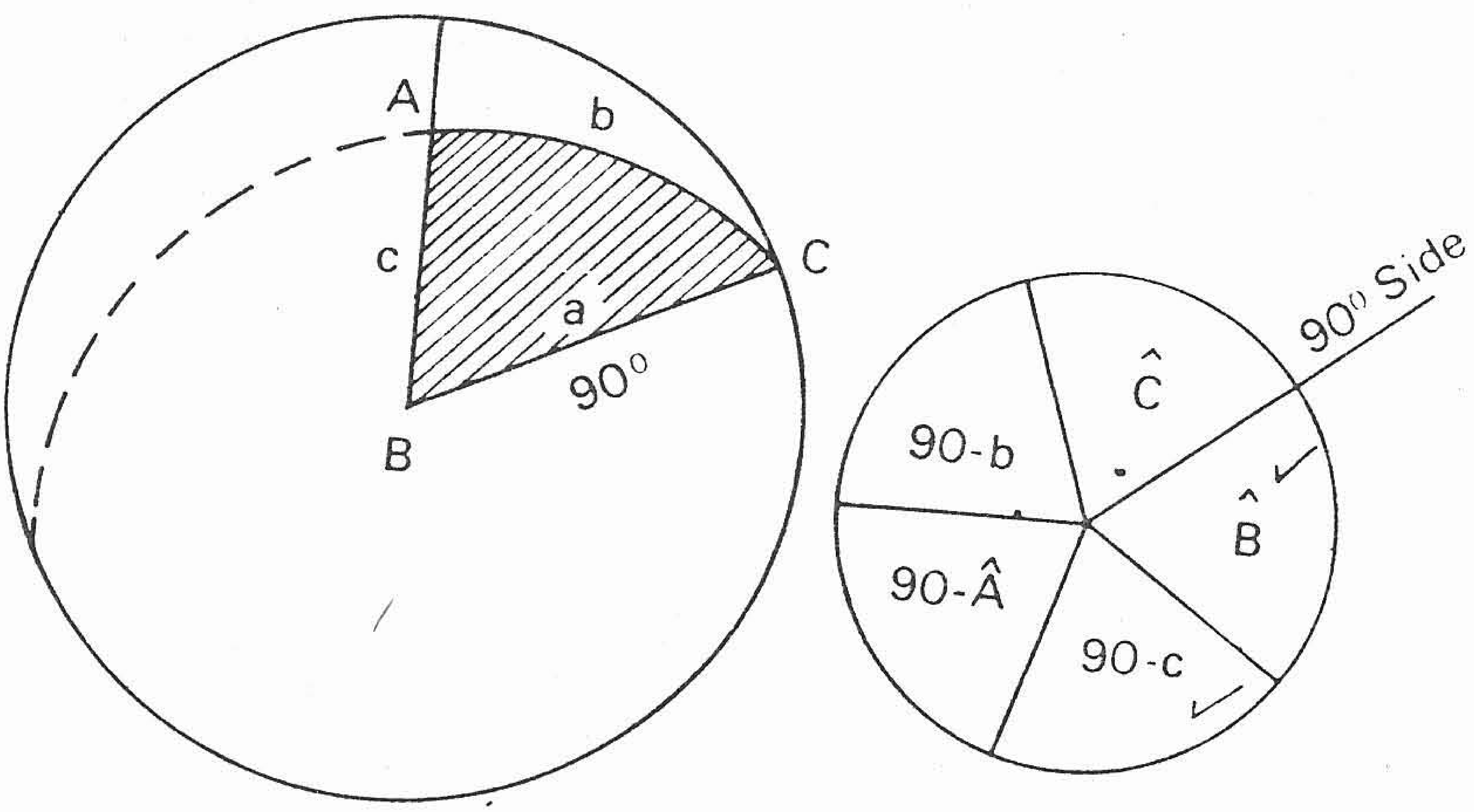


Fig. 76

Example

If in △ A B C in fig. 76, side c & $\hat{B}$ are given and side BC is 90° (Quadrantal side), to find the remaining parts ie. $\hat{A}$, sides b & c.

We go about as follows :-

Tick (✓) off the known parts in the circle, then use the Napier's Rules.

To find $\hat{C}$

Sin B = tan C tan (90 - c) (Rule 1)

tan C = Sin B tan c

Now use logs. for solving $\hat{C}$

To find side b

$\sin(90 - b) = \text{Cos } B \ \text{Cos}(90 - c)$ (Rule 2)

$\text{Cos } b = \text{Cos } B \ \sin c$

To find $\hat{A}$

$\text{Sin}(90 - c) = \tan B \ \tan(90 - A)$ (Rule 1)

$\text{Cot } A = \text{Cos } c \ \cot B$

The Rules regarding "Like & Unlike affection" etc apply here also. The only difference is that the angle oppoisite to the quadrantal side is **more than 90°**, if the two adjacent sides are of **unlike affection.**

When using "Rule of Signs", insert a minus (–) sign to the product if two adjacent or opposite parts are both angles or both sides. This will then determine whether the required value is in the first or second quadrant.

Worked example.

In a quadrantal triangle PQR given p = 90°, $\hat{P}$ = 70° and $\hat{R}$ = 55° find sides 'q' & 'r' and $\hat{Q}$.

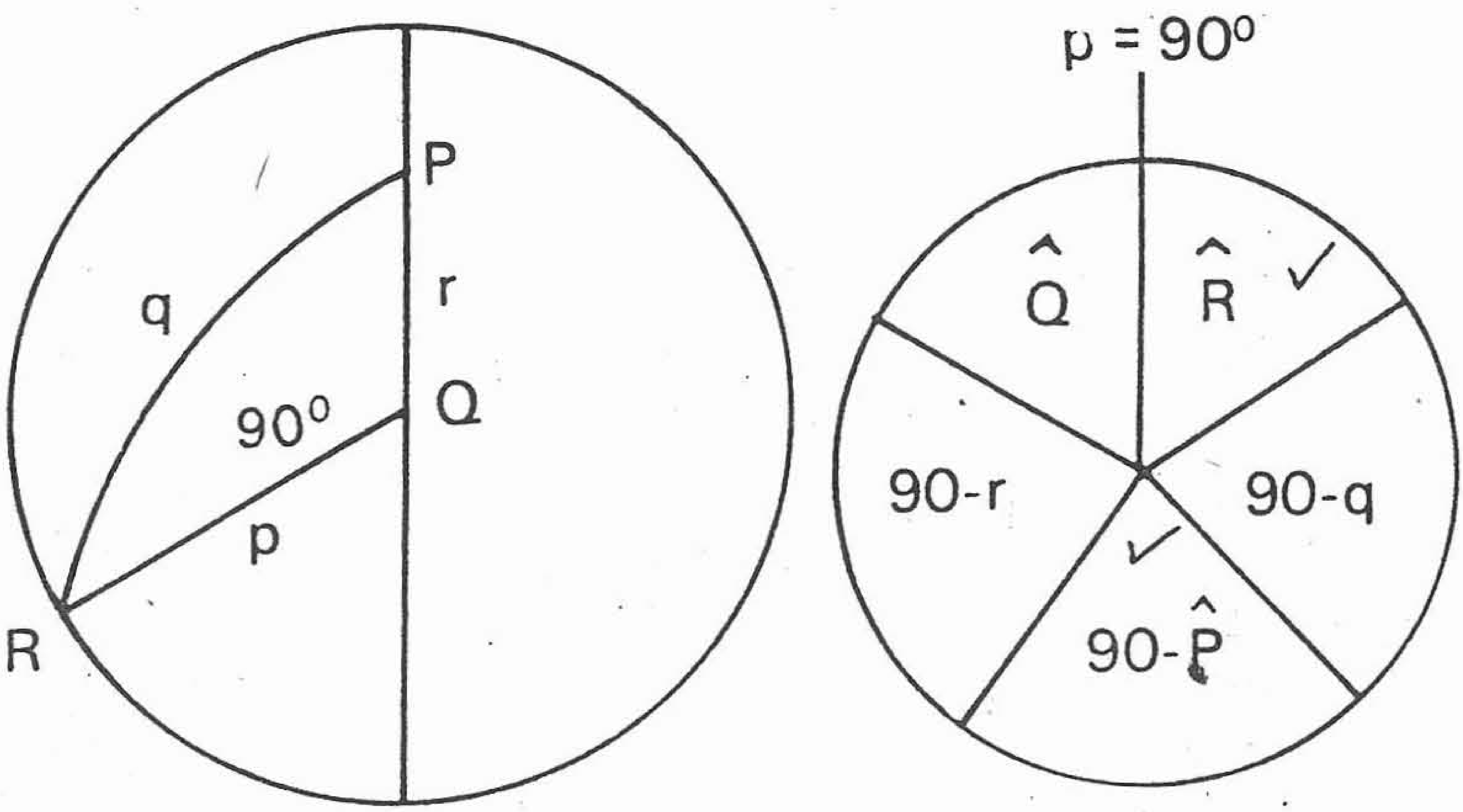


Fig. 77

To find side 'r'

$\text{Sin } R = \text{Cos}(90 - r) \ \text{Cos}(90 - P)$

$\text{Sin } R = \text{Sin } r \ \sin P$

$\text{Sin } 'r' = \text{Sin } R \ \text{Cosec } P$

$= \text{Sin } 55 \ \text{Cosec } 70°$

No.	Rat.	Log.
55	Sin	9.91337
70	Cosec	0.02701
60° 39½	Sin	9.94038

The Rule of signs will fail in this case because sin & cosec are both + ve in the second quadrant. Since $\hat{R}$ is less than 90° side 'r' is also less than 90°, as the angle and the side opposite to it are always of same affection.

Side 'r' = 60° 39½'

To find $\hat{Q}$

Sin (90 – P) = Cos Q Cos R

Cos P = Cos Q Cos R

(–) Cos Q = Cos P Sec R

Put a – ve sign in front of product because right hand side are both angles. By doing this we are employing the "Rule of Signs".

No.	Rat.	Log.	
70°	Cos	9.53405	180° 00'–
55°	Sec	0.24141	53° 24'
126° 36'	(–) Cos	9.77546	126° 36'

$\hat{Q}$ = 126° 36'

To find side 'q'

Sin (90 – q) = tan R tan (90 – P)

(–) Cos q = tan R Cot P

Here also enter a (–) sign in front of the product because the two values on Right hand side of the equation are both angles. By doing this we know that the value of 'q' must be more than 90°. This will also fit in with the "rule of affection", which states that an angle & the side opposite to it are always of like affection. Since $\hat{Q}$ is more than 90°, side 'q' also has to be more than 90°.

No.	Rat.	Log.	
50°	Tan	0.15477	180° 00'–
70	Cot	9.56107	58° 41'
121° 19'	(–) Cos	9.71584	121 19'

Side 'q' = 121° 19'

Applications of right angled or quadrantal spherical triangles do arise from time to time for solving navigational problems particularly in the examinations. Some of the applications are illustrated below.

(1) A Body on prime vertical (Fig. 78)

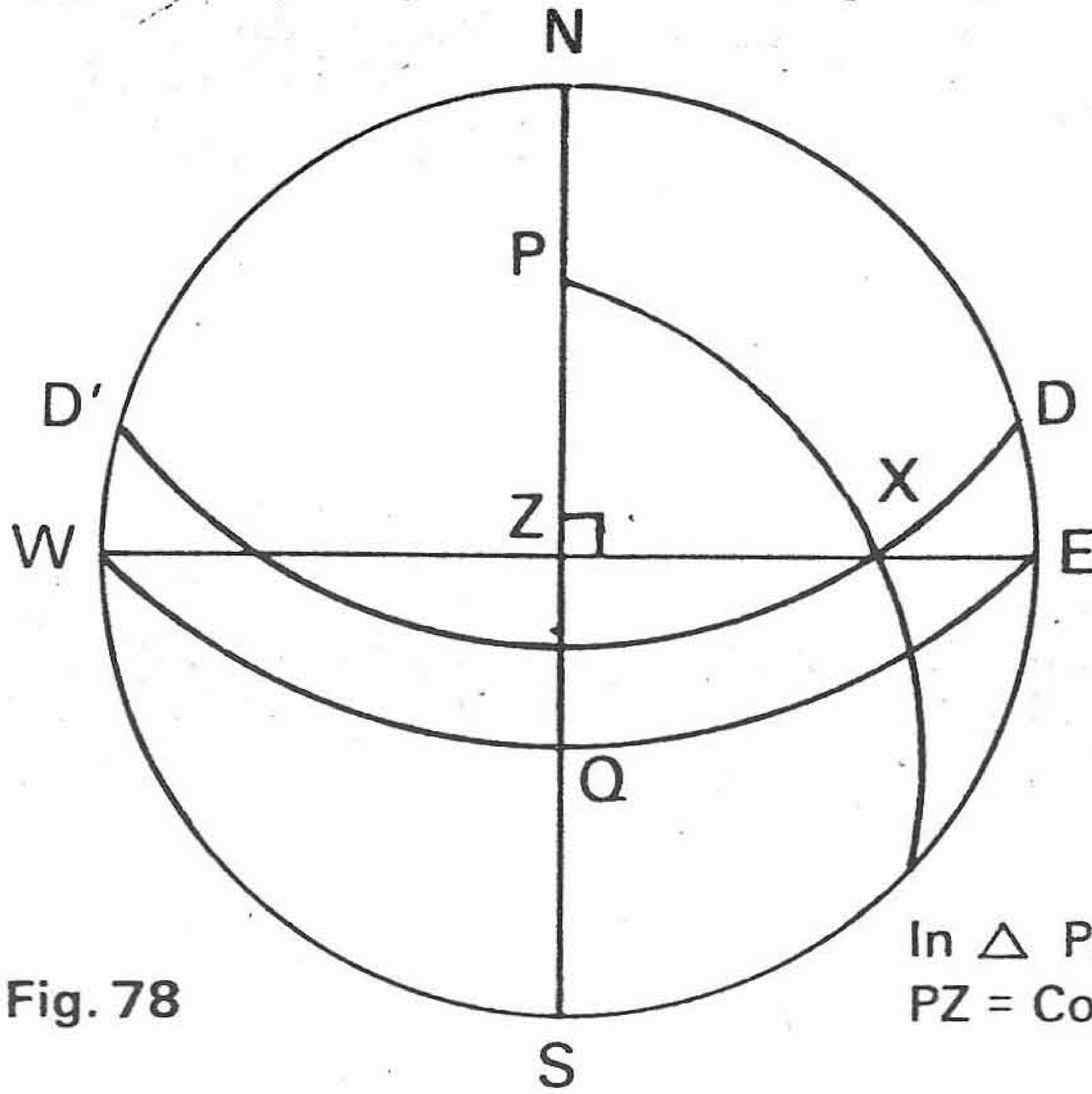


Fig. 78

In △ PZX, PX = Polar dist.
PZ = Co–Lat., $\hat{Z} = \mathbf{90°}$

(2) **A body at its Maximum-Azimuth** (fig. 79)

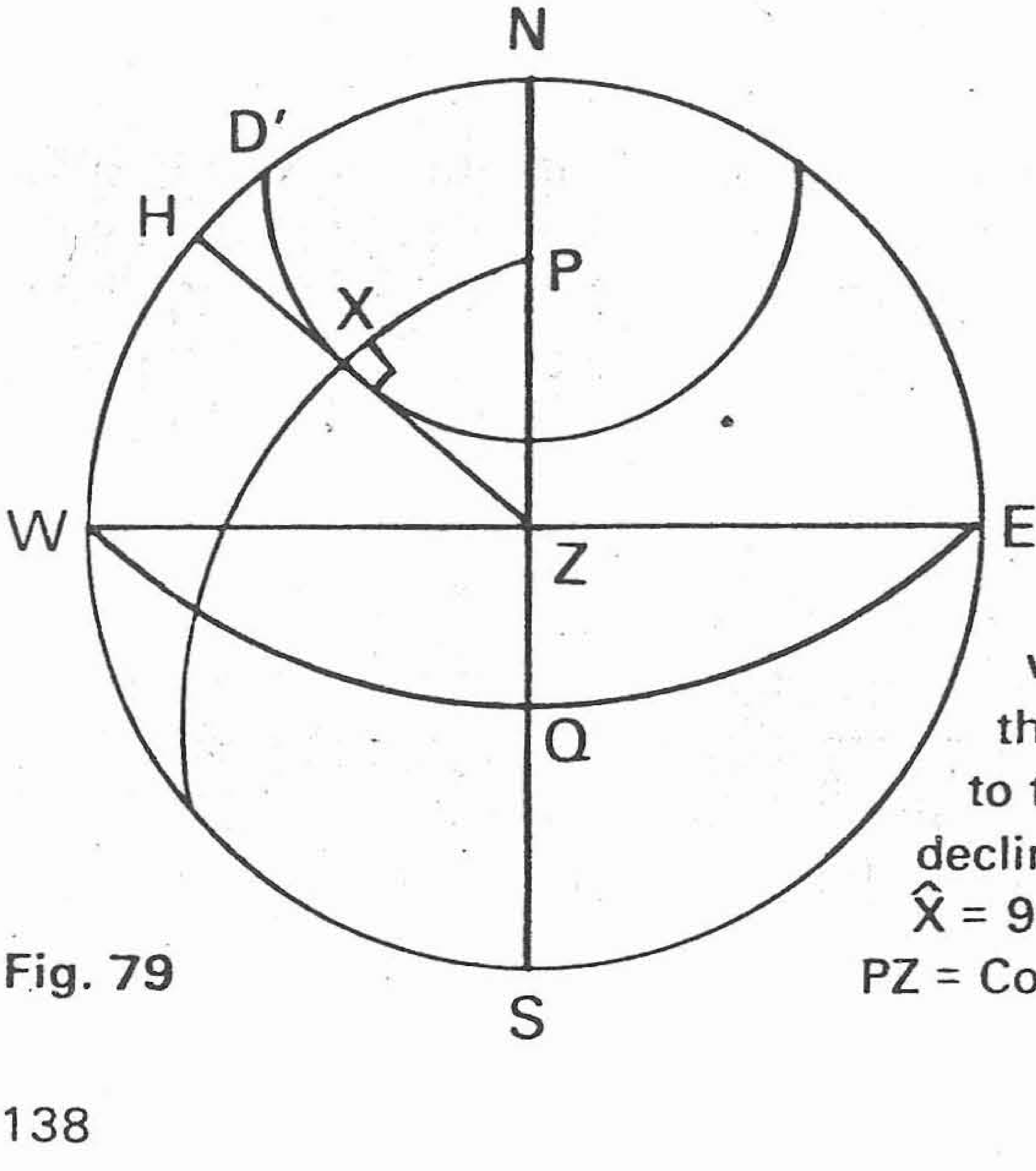


Fig. 79

In △ PZX ;
X is body at its max. Azimuth ie. when the vertical circle through the body is tangential to the parallel of declination of the body.
$\hat{X} = 90°$, PX = Polar Dist.
PZ = Co–Lat.

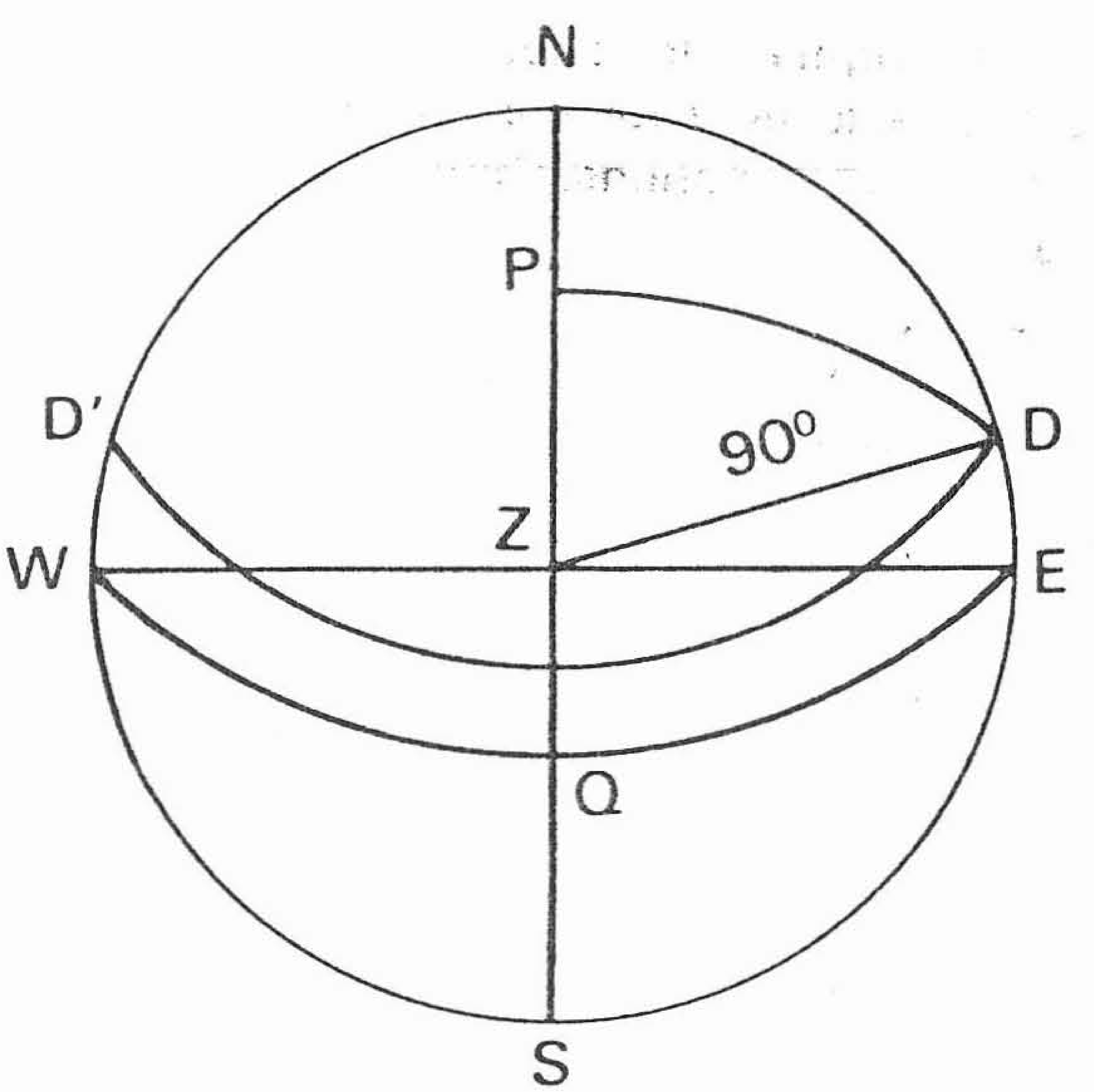


Fig. 80

(3) **A body at Rising/Setting** (fig. 80)
D is the body on the rational horizon at rising. It will be at D' at setting
In △ PZD
PD = Polar dist.
PZ = Co-Lat.
ZD = 90° (Quadrantal side)

III Solution of Oblique Spherical Triangles

Spherical triangles other than right angled or quadrantal triangles are oblique spherical triangles. Unlike a right angled or quadrantal triangle, it is necessary to know any three parts, to find the remaining three parts of an oblique spherical triangle. Position finding at sea, and great circle sailing are both dependent on solution of oblique spherical triangles.

There are several ways of solving these triangles.

The fundamental formula for solution of all spherical triangles is the **Cosine formula.** viz.

Cos a = Cos b Cos c + Sin b Sin c Cos A.

The most common method adopted by mariners is the use of **Haversine Formula.** The main advantage in using this formula is that Haversine values of all angles upto 180° is + ve.

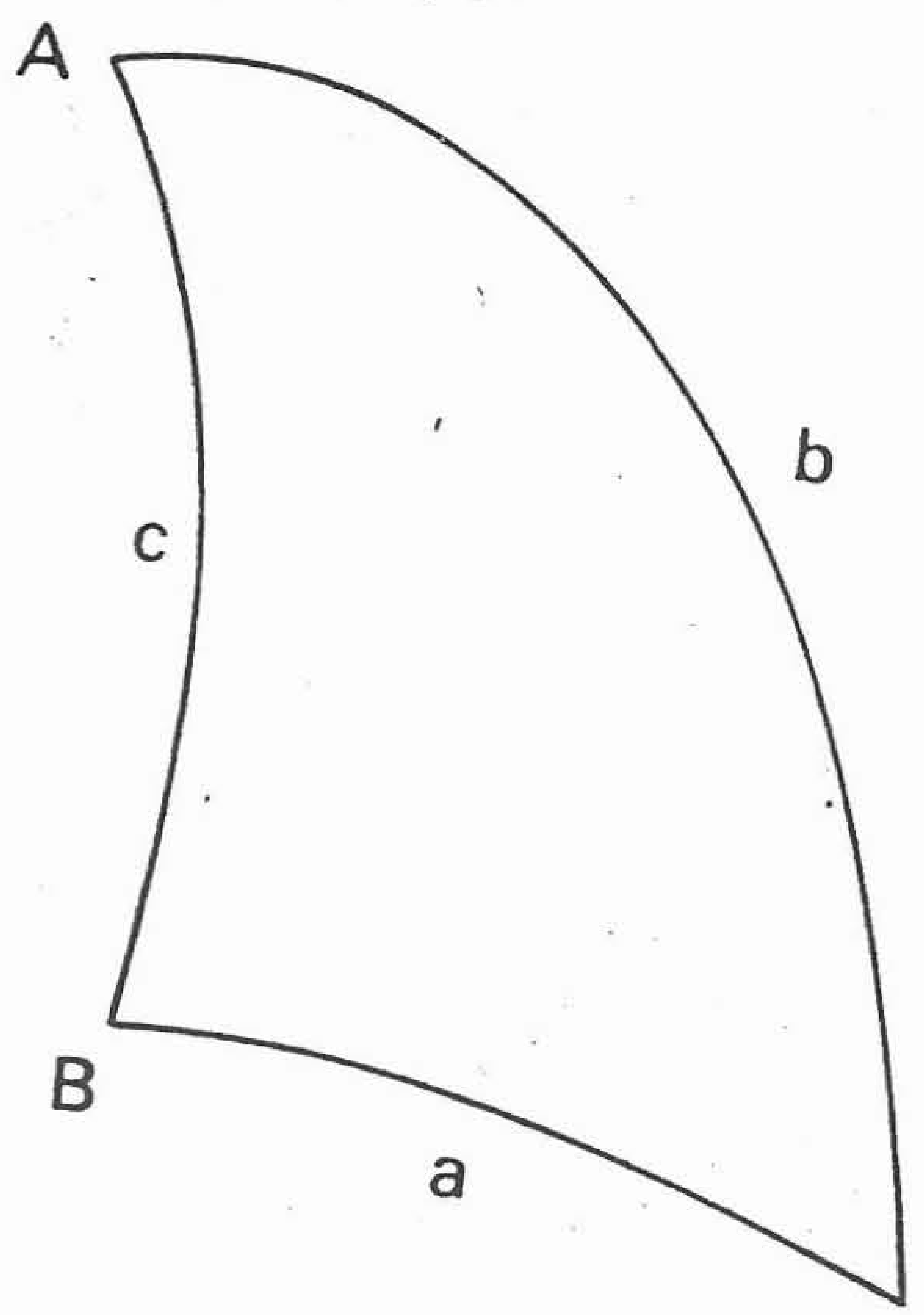


Fig. 81

In a spherical triangle (fig. 81)

(i) **Given the three sides** a, b & c, to find all three angles.

Hav A = [Hav a – hav (b~c)] Cosec b cosec c

Hav B = [Hav b – hav (a~c)] Cosec a cosec c

Hav C = [Hav c – hav (a~b)] Cosec a cosec b

(ii) **Given two sides and the included angle**

In fig. 81, if $\hat{A}$, and side b & c are given to find, side 'a', $\hat{B}$ & $\hat{C}$

Hav a = [Hav A Sin b sin c + Hav (b~c)]

This formula is also used in finding the great circle distance along a G.C. track. Having found 'a' we can apply the formula stated in section (i) above viz :

$$Hav\ B = \left[Hav\ b - Hav\ (a \sim c)\right] Cosec\ a\ Cosec\ c$$

$$Hav\ C = \left[Hav\ c - Hav\ (a \sim b)\right] Cosec\ a\ cosec\ b$$

This can also be solved by use of **Napier's Analogies.**

$$Tan\ \tfrac{1}{2}\ (B+C) = \frac{Cos\ \tfrac{1}{2}\ (b \sim c)}{Cos\ \tfrac{1}{2}\ (b+c)}\ Cot\ \tfrac{1}{2}\ A$$

$$Tan\ \tfrac{1}{2}\ (B \sim C) = \frac{Sin\ \tfrac{1}{2}\ (b \sim c)}{Sin\ \tfrac{1}{2}\ (b+c)}\ Cot\ \tfrac{1}{2}\ A$$

(iii) Of the two angles and two sides opposite to each other, if any three are given, then the fourth can be found by use of the **Sine Formula.**

$$\frac{Sin\ A}{Sin\ a} = \frac{Sin\ B}{Sin\ b} = \frac{Sin\ C}{Sin\ c}$$

(iv) Any four adjacent parts of a spherical triangle is connected by the **"Four Part Formula"**

$$Cot\ C\ Sin\ A = Cot\ c\ sin\ b - Cos\ A\ Cos\ b.$$

The ABC azimuth tables in Norie's & other nautical tables are based on this formula.

(v) If a perpendicular is drawn from an angle to the opposite side, this perpendicular divides the obilque triangle into two right angled triangles (see fig. 82). BD is a perpendicular drawn from B to side AC. The two right angled triangles ABD and BCD can now be solved to find the values of all the required parts of $\triangle$ ABC.

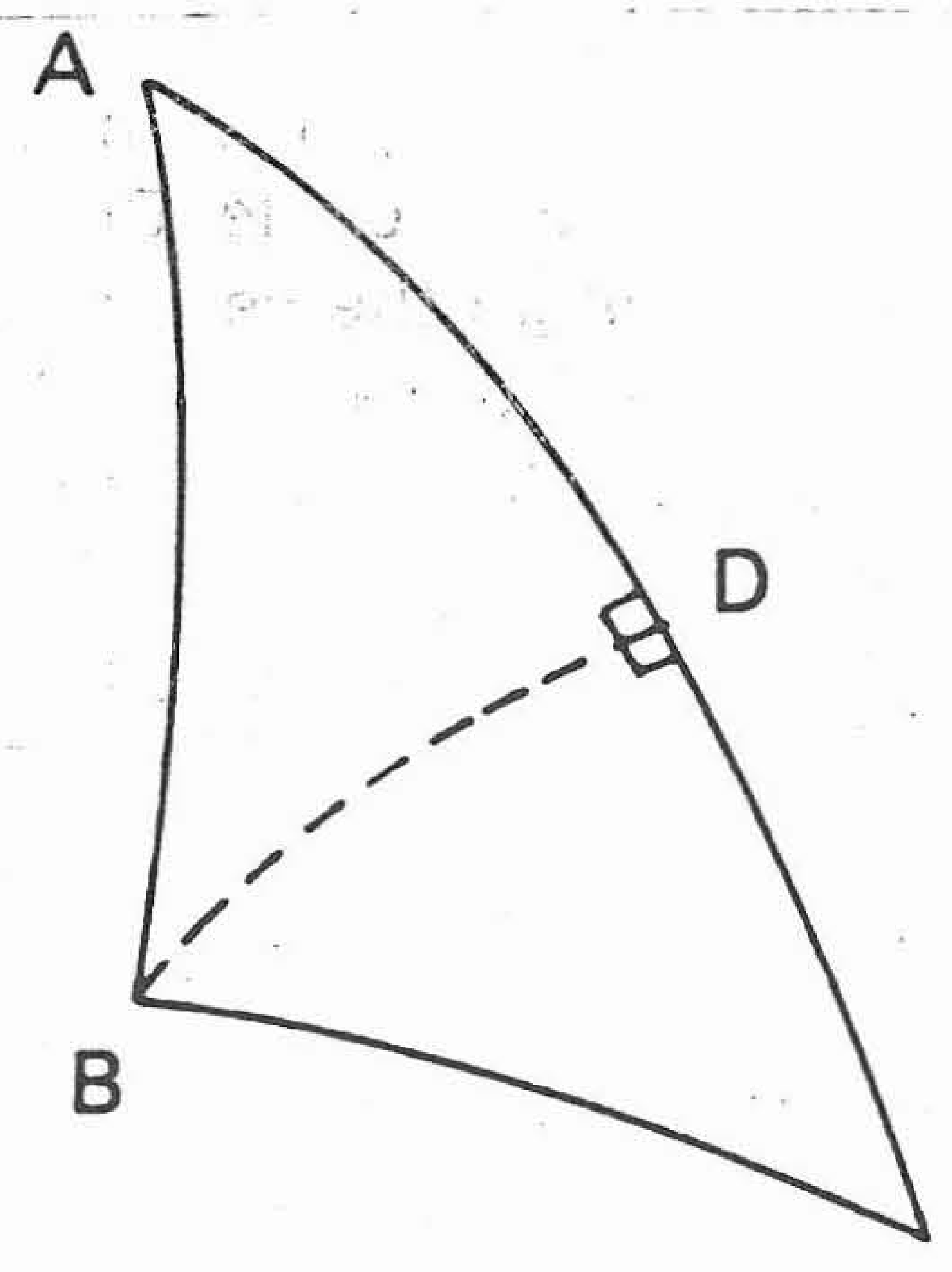


Fig. 82

GREAT CIRCLE SAILING

The lesser arc of Great Circle being the shortest distance between any two points on a sphere, G.C. sailing is often used on the earth's surface, particularly for inter–continental ocean crossing, which involves large distances. Unlike a rhumb line, a G.C. does not cross all meridians at the same angle. Hence when sailing along the G.C. it is necessary to continously change the ship's course in order to head directly for the destination. Since it is not practical to continously alter the course of the ship along the track, a number of points, a few degrees of longitude apart are chosen along the G.C. track and the ship sails along rhumb line courses between these points. By doing this, she is sailing very nearly along a G.C. track, though not exactly along the track.

Worked Example : Find the intial & final courses and the G.C. distance and position of the vertex on a G.C. track from a position 'A' in latitude 37°N long. 140°W to a position 'B' in lat. 52°N long 160°E.

Construction of figure (fig. 83)

It is always desireable to draw a fairly accurate figure in order to see the relative positions of the departure and arrival positions, and position of vertex etc.

1 It is generally drawn on equidistant projection. Select any convenient radius and draw a circle. Draw the NS & EW line at right angles to pass through the centre Q – NS represents the central meridian and WQE the equator. Draw a line QA making angle EQA = 37°, to place 'A', the lower of the two latitudes on the circumference. The arc NAES then represents the meridian of A = 140° W. Mark an angle WQL' = 52° = Latitude of B. Draw a perpendicular to QL' at L' to meet NS produced at P. With P as centre and PL' as radius draw an arc L'L to represent latitude of B. Make an angle at N equal to D' long between A & B (60° in this case) and let this line meet EW produced at some point K. With K as centre and radius equal to KN draw the arc NS meeting LL' at B. Now B represents the position of the destination. (If the D'long between A & B exceeds 90° then take the supplement of D'long and mark the angle at N on the same side as A to find point K). Produce AQ to D to form a diameter & draw QR right angles to AD at Q. Draw the right bisector of AB to meet the

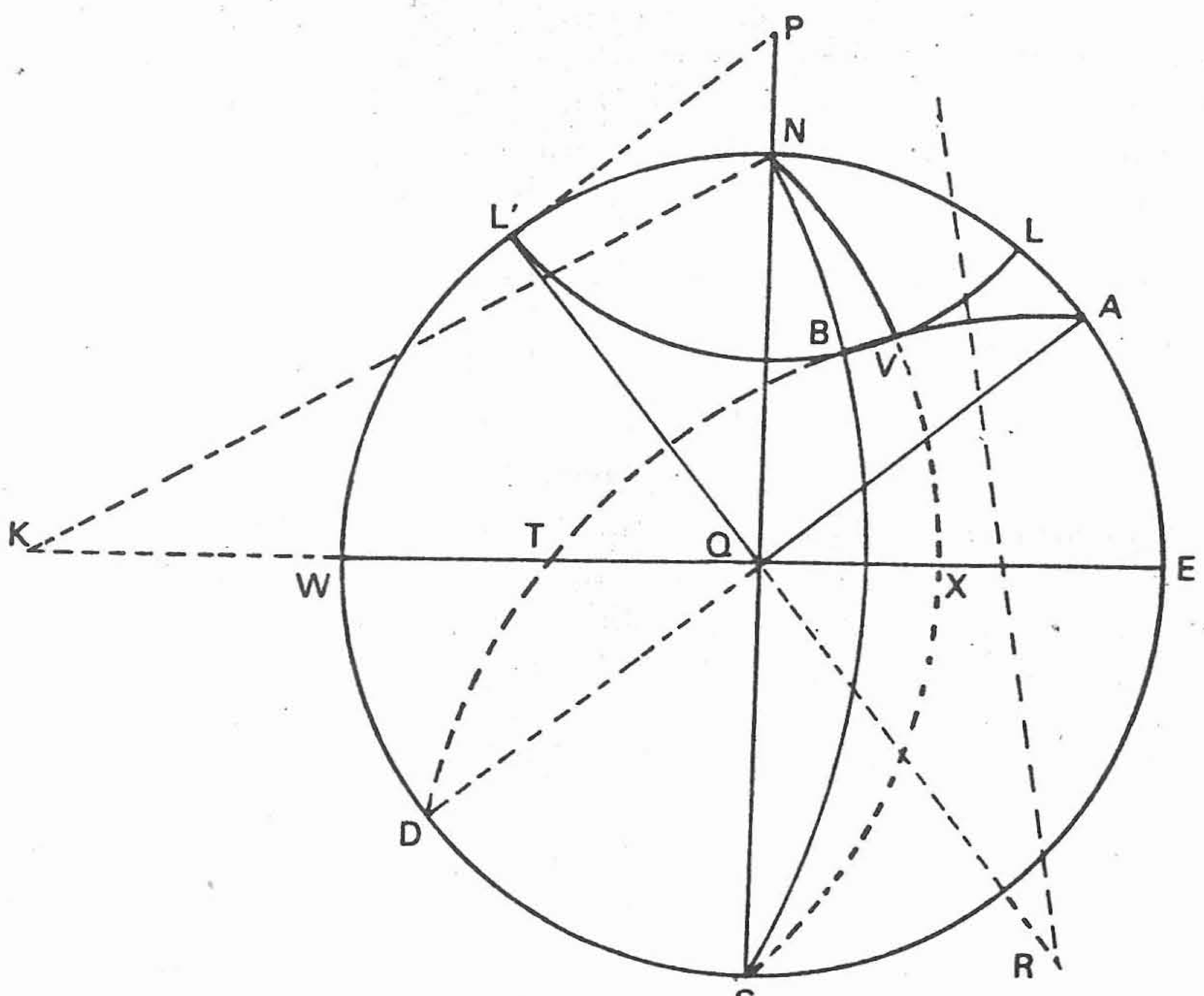


Fig. 83

perpendicular to AD at R. With R as centre and radius equal to RA or RD draw the arc DBA. Now AB is the G.C. track between A & B.

This G.C. crosses the Equator at 'T'. With T as centre and radius used for the primary circle representing 90° mark off a point X on WQE. By trial and error draw a meridian passing through X to meet the G.C. at V. Point V represents the Vertex of G.C. The centre of this meridian will lie along QW or QW produced.

Vertex is the turning point on a G.C. It represents the highest Latitude reached along that G.C. The course at the vertex is always E/W. This would mean that the angle between the meridian of the vertex and the G.C. track at V is always a Right angle.

Point T on the Equator being the Pole of the meridian passing through V, the course of the ship on the G.C. if and when she crosses point T will always be equal to the Co-lat of the Vertex.

In triangle NAB given

NA = Co-lat of A = 90-37° = 53° = side 'b'
NB = Co-lat of B = 90-52° = 38° = side 'a'
$A\hat{N}B$ = D'long between A & B = 60° (West)

To find :- (i) G.C. Distance : AB
(ii) Initial course : $N\hat{A}B$
(iii) Final Course : $N\hat{B}A$
(iv) Lat & Long of vertex,
given angle V = 90°

To find G.C. distance

Hav n = Hav N sin a sin b + Hav (a~b)

No.	Rat.	Log.	Nat.
N = 60°	Hav	9.39794	
a = 38°	Sin	9.78934	
b = 53°	sin	9.90235	
		9.08963	0.12284
a~b = 15°	Hav		0.01704
n = 43° 55½'	Hav		0.13988

G.C. dist. = 43° 55½' = 2635½ miles

To find initial course :

Hav A = [Hav a − Hav (n ~ b)] Cosec n Cosec b

	No.	Rat.	Log.	Nat.
a	= 38°	Hav		0.10599 −
n~b	= 9° 4½′	Hav		0.00626
			8.99884	0.09973
n	= 43° 55½′	Cosec	0.15882	
b	= 53°	Cosec	0.09765	
A	= 50° 12.6′	Hav	9.25531	

Initial course = N 50° 12.6′ W (309° 47.4′)

To find final course :

Hav B = [Hav b − Hav (n~a)] Cosec n Cosec a

	No.	Rat.	Log.	Nat.
b	= 53°	Hav		0.19909 −
n~a	= 5° 55½′	Hav		0.00267
			9.29318	0.19642
n	= 43° 55½	Cosec	0.15882	
a	= 38°	Cosec	0.21066	
B	= 85° 24′	Hav	9.66266	

Final course = S 85° 24′ W (265° 24)

Note that initial course is N & W whereas the final course is S & W. This is because the vertex lies between A & B and the course therefore changes from northwards to sourthwards on crossing the vertex. If vertex was outside AB, then initial & final courses would have the same names.

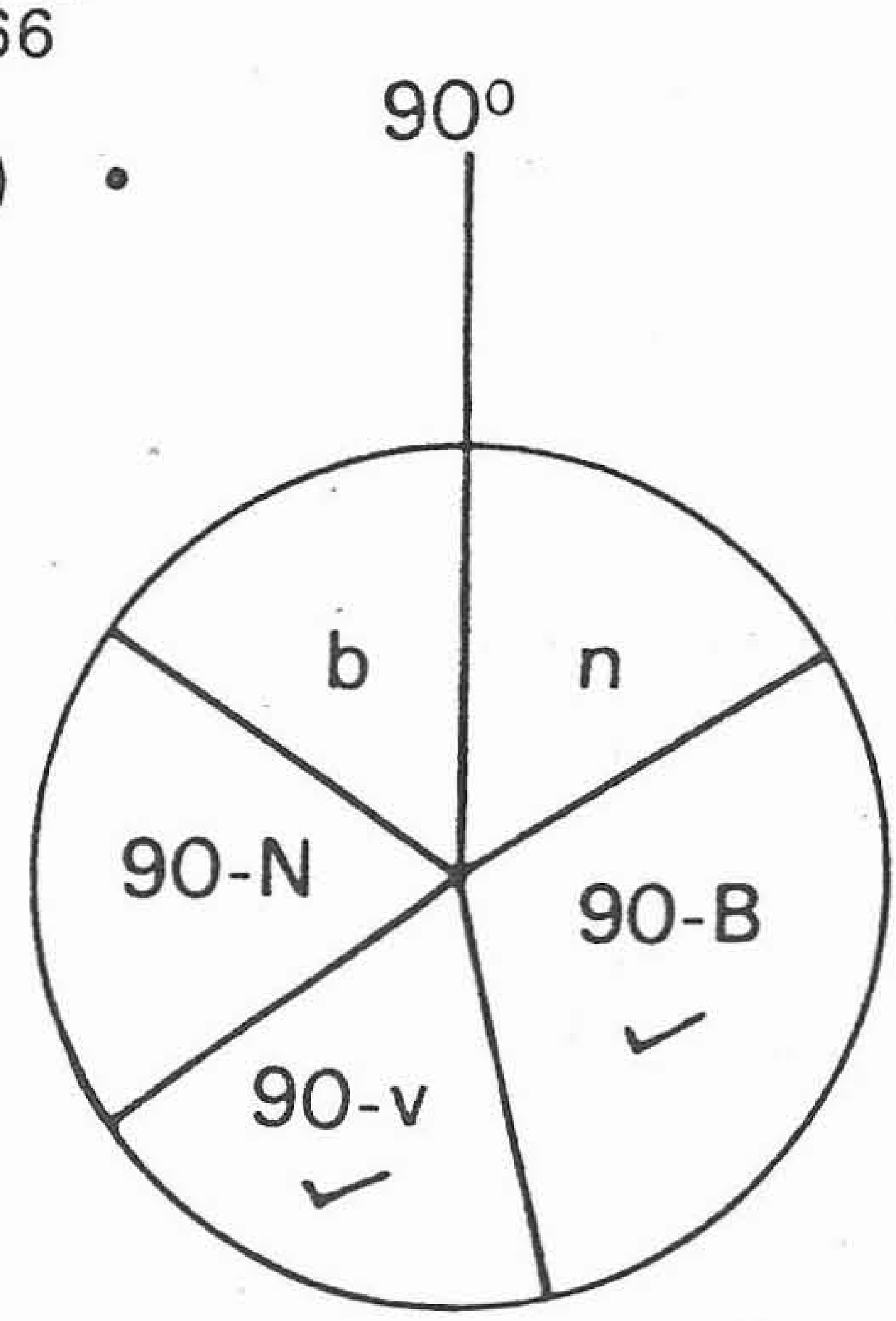


Fig. 84

The initial & final courses could have been simultaneously found out by use of the tangent formula (Napier's Analogies.)

To find the position of Vertex :

In$\triangle$NBV (fig. 83), given

$\hat{B}$ = Final course = 85° 24'
NB = v = Co-lat of B = 38°
$\hat{V}$ = 90°

To find b = Co-lat of V and thence Lat of V.
$\hat{N}$ = D'Long between B & V and thence Long of V.

Using Napier's Rules

Sin b = Cos (90 – v) Cos (90 – B)
Sin b = Sin v Sin B.

No.	Rat.	Log.
38°	sin	9.78934
85° 24'	sin	9.99860
37° 51½'	sin	9.78794

Co Lat of V = 37° 51½'
Lat of V = 52° 08½ N.

Sin (90 – v) = tan (90 – B) tan (90 – N)
Cos v = Cot B Cot N.

∴ Cot N = Cos v Tan B.

No.	Rat.	Log.
38°	cos	9.89653
85° 24'	tan	1.09443
5° 50'	cot	0.99096

Long of B	=	160° 00' E
D'long	=	5° 50' E
Long of V	=	165° 50' E

Answers :-	G.C. dist.	– 2635½ miles
	Initial co	= N 50° 12.6 W
	Final Co	= S 85° 24' W

Pos. of vertex = Lat 52° 8½N Long 165° 50' E

As stated earlier in this section, it is necessary to select convenient points along the track to facilitate the sailing along the G.C. Choose convenient longitudes, say every 5 or 10 degree apart. The triangles formed by the meridian of the vertex, the selected longitudes and the G.C. track will form series of right angled triangles in which co–lat of V is known, the angle at 'N' is known, being the D'long between Long of V & the chosen Longitude and angle at V = 90°. It is required to find the co–lat of the chosen points & thence its latitudes.

This can be easily worked out by use of Napier's Rules for circular parts.

Once the Latitudes and Longitudes of several such points are known on the G.C. track, the ship sails along short legs of rhumb lines from point to point. Thus following the Great circle as close as possible.

Wireless bearing

When a D/F bearing of a distant station is taken the bearing we obtain is a Great circle bearing. The wireless waves take the shortest path along the curvature of the earth which is a G.C. We cannot plot this directly on the chart.

The G.C. bearing will have to be converted to Mercatorial bearing before it can be plotted on the mercator's chart.

The Correction we apply to the G.C. bearings for this conversion is called the **"Half Convergency"**.

$$\text{Half convergency} = \frac{\text{D. Long x sine Mean Lat.}}{2}$$

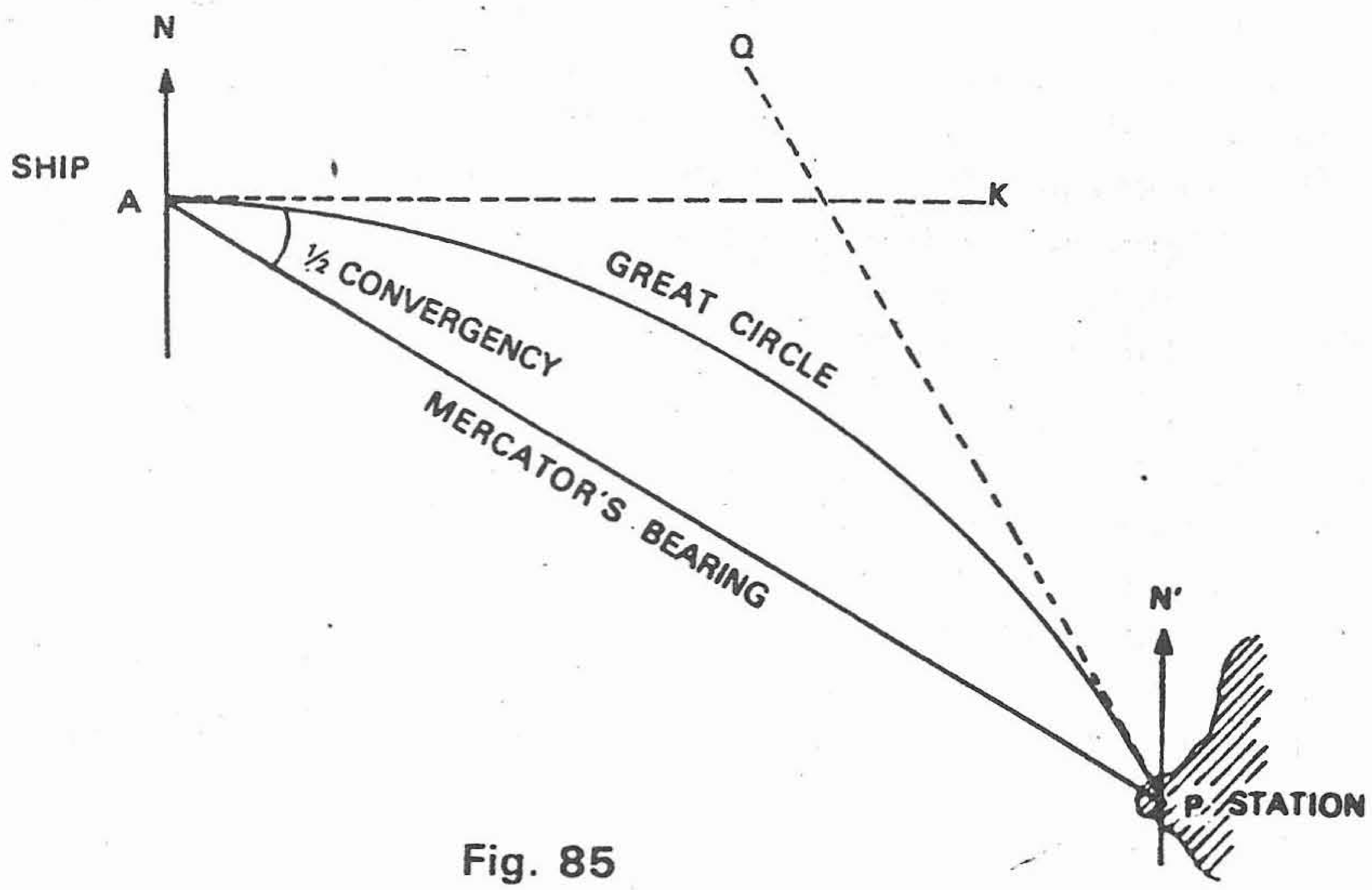


Fig. 85

If a ship at A takes a D/F bearing of a Radio Beacon placed at point P on the chart, the W/T wave travels along the G.C. arc P A To the ship at A, the bearing will ie. along the tangent to the G.C. at A ie. the bearing obtained on his D/F set will be $N\hat{A}K$.

If, on the other hand, point P were to take the bearing of the ship at A, the bearing indicated at the shore station will be $N'\hat{P}Q$. The straight line joining AP is the Mercatorial Bearing, which can be drawn on the chart.

The half convergency correction is the angle KAP at the ship or QPA at the shore station. This can be calculated once the DR position of the ship and the position of the shore station are known. D'long between ship & shore station can be calculated and also the Mean Latitude between the two points.

$\dfrac{\text{D'long x sine Mean Lat}}{2}$ will then give half convergency.

Rule for applying the correction

The convex part of the G.C. always lies facing the pole of the hemisphere in which the G.C. is. Therefore the mercatorial bearing always lies on the Equitorial side of the G.C. Hence the half convergency correction is always applied towards the equator.

The convert the G.C. bearing of point P, taken by ship A, the half convergency correction will be added to make it the mercatorial bearing. If the shore station were to take the bearing of the ship, then this correction has to be subtracted to arrive at the mercatorial bearing. This is quite evident from the figure 85. The sign of the correction can very easily be judged by drawing a rough sketch of the relative positions of the ship and the station. fig. 85 represents a G.C. bearing of the ship & station placed in the Northern hemisphere.

In the Southern hemisphere, the convex part of the G.C. itself will face southwards. So a correction which is additive in the northern hemisphere will be subtractive in southern hemisphere and vice versa. Simplest way to accertain the sign of the correction, is to always apply it towards the equator. In the Northern hemisphere the equator lies to the South of the observer and in the Southeren hemisphere the equator lies to the North of the observer.

Worked example

A ship in DR position 46° N 9° W and steering a course of 316° (T) takes a D/F bearing of shore station in Lat 50°N 6°W and obtains the D/F bearing as 078° Relative. Find the mercatorial bearing.

Ship's head	= 316° (T)
Relative Brg.	= 078°
D/F Bearing	= 394°
	− 360
	= 034° (T)

Ship DR	Lat 46° N	Long 9° W
Stn Pos.	Lat 50° N	Long 6° W
	D'Lat 4° N	D'Long 3° E

Mean Lat = 48° N

$$\text{Half convergency} = \frac{3x^\circ \sin 48^\circ}{2}$$
$$= 1.5^\circ \times \sin 48^\circ$$
$$= 1.1^\circ$$

It is evident from this rough sketch (fig 86) that the correction is + ve

D.F. Bearing	= 034° (T)
Half convergency	= + 1.1°
Mercatorial Brg.	= 035.1° (T)

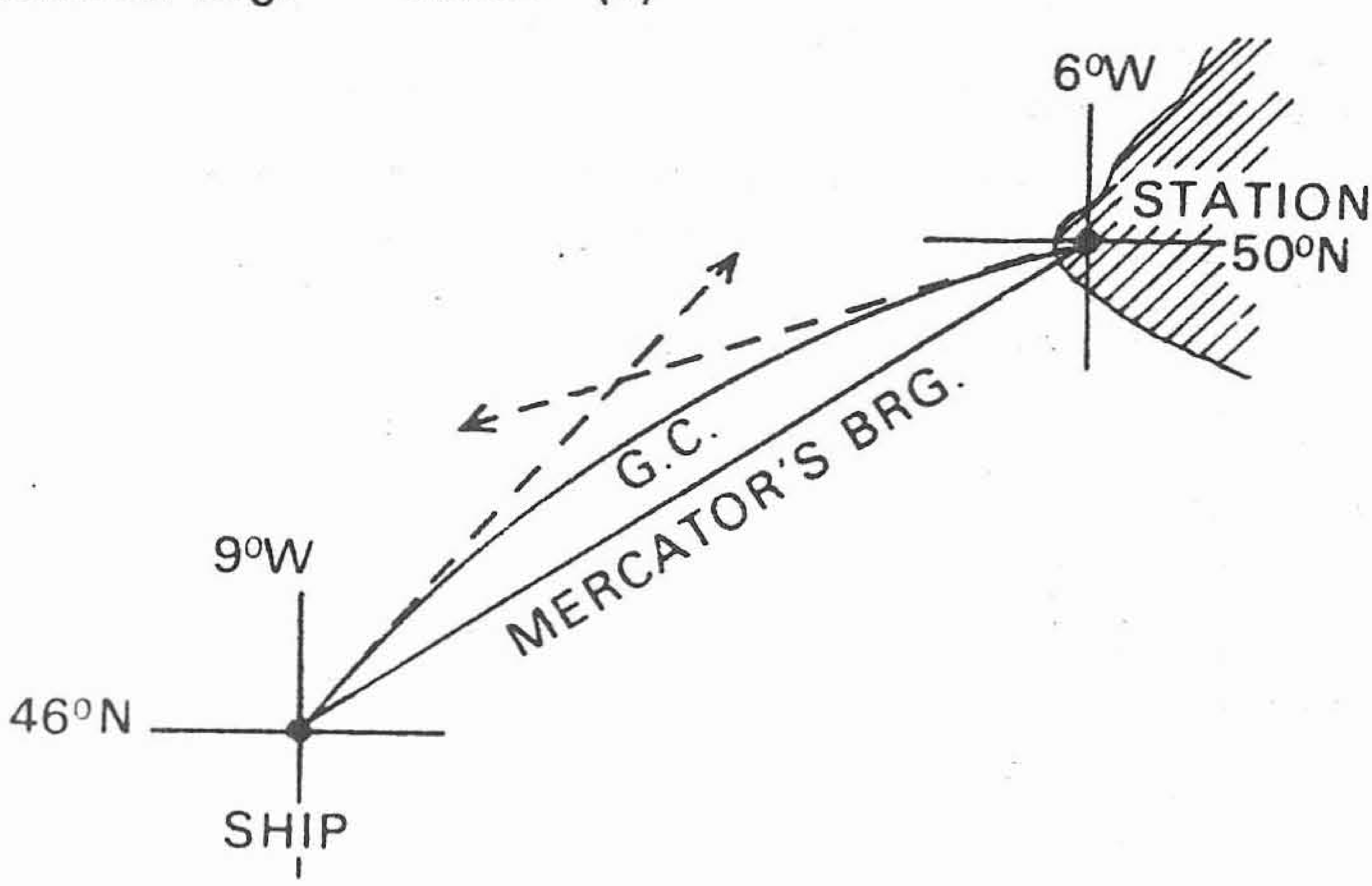


Fig. 86

EXERCISE XIII

(1) How is a spherical triangle formed ? What are the properties of a spherical triangle ?

(2) What is meant by the terms "Like or Unlike Affection". State the three axioms based on these said affections.

(3) What is the advantage of Great Circle sailing over Mercator's sailing ? Under what dcircumstances are the advantages most beneficlal ?

(4) Can a ship sail exactly over a G.C. Track ? If not, why ? In practice how is the G.C. track made use of ?

(5) In a Right angled spherical triangle ABC, given the following data, find the remaining 3 parts :

(a) $\hat{B} = 90°$ $a = 22° 18'$ $c = 45° 35'$

(b) $\hat{B} = 90°$ $\hat{A} = 44° 20'$ $b = 54° 10'$

(c) $\hat{B} = 90°$ $\hat{C} = 71° 05'$ $a = 49° 30'$

(6) Given Decl. of Sun is 21° 51'S, and its S.H.A. is 67° 37.8' find the obliquity of the ecliptic.

(7) In a quadrantal triangle ABC, given, $\hat{A} = 67° 02'$, $c = 46° 00'$ and $a = 90°$, find the remaining three parts.

(8) When the decl. of sun was 15° 30' S, its Azimuth was 253° 11' at sunset, Find the latitude of the observer.

(9) In a lat. 49° 10'N at Sunrise, sun's decl. was 23° 26'N and its GHA was 286° 6.0'. Find the longitude of the observer.

(10) In an oblique spherical triangle ABC, given $a = 65°$, $b = 84°$, $c = 92°$, find the three angles.

(11) In $\triangle$ ABC given, $b = 54° 30'$, $c = 103° 40'$ $\hat{A} = 48° 45'$, find the remaining three parts.

(12) In $\triangle$PZX given PZ = 41°, PX = 67°, $\hat{P} = 28°$ find ZX.

(13) Find the initial and final courses and distance along a G.C. Track and the position of vertex, from a position 'A'. Lat. 56°S Long. 67°W to position 'B' in Lat. 37°S, Long 19°E.

(14) From Hawai in Lat. 21° 30'N Long. 157° 50'W to Vancouver in Lat. 48° 20'N, Long 123° 15'W. Find G.C. dist. initial and final courses and the position of vertex.

(15) From 'A' in Lat. 34° 00'S, Long 25° 30' E to B in Lat. 5° 50'N, 80° 35'E, find G.C. dist. initial and final courses and position of vertex.

(16) A ship is DR Lat. 23° 10'N, Long. 123° 18'E, observes the D/F bearing of a shore station in Lat. 22° 12'N. Long 114° 20'E to be 266° (T). Find the mercatorial bearing to be plotted on chart.

(17) A Vessel steering 085° (T) in DR Lat. 41° 50'N Long 67° 50'W takes a D/F bearing of a shore station in Lat. 44° 40'N Long 63° 35'W. If the D/F relative bearing was 323½° find the mercatorial bearing.

Answers

Q. 5. (a) A = 29° 42'   C = 69° 42½'   b = 49° 36'

(b) a = 34° 30½'   c = 27° 19'   C = 30° 56'

(c) A = 52° 05½'   b = 74° 31½   c = 65° 44½'

(6) 23° 26½'   (7) B = 170° 41½', b = 112° C = 41° 28½'

(8) 22° 31½S   (9) 46° 12'W

(10) A = 64° 36.2' B = 82° 26' C = 95° 02'

(11) a = 67° 23.7' B = 41° 29' C = 127° 39'

(12) 34° 5.7'

(13) G.C. dist 3479.3' Initial Co. S 69° 58.8'E (110° 01.2')
Final Co. N 41° 08.3'E (041° 08.3')
Lat of V = 58° 18.2'S Long of V = 00° 43½'W

(14) G.C. Dist = 2307.7' Initial Co. N 37° 21'E (037° 21')
Final Co.N 58° 06.6'E (058° 06.6')
Lat of V = 55° 38'N Long. 83° 28½'W

(15) G.C. Dist. 3927.9' Inital Co. N 63° 43.7'E (063° 43.7)
Final Co.N 48° 21.3E (0.48° 21.3')
Lat of V = 41° 59½'S Long V = 23° 03.8'W

(16) ½ Convegency = −1.7° Mercatorial Brg 264.3° (T)

(17) ½ Convergency= + 2° Mercatorial Brg 050½°

TEST PAPERS

Each of the following papers are intended to be worked out in 2 Hours and carries 100 marks. These are based on the proposed new syllabus. Any reference to Nautical almanac is not required as necessary data is given in the question itself. In examinations however the data may have to be taken from the Nautical Alamanac for the appropriate year.

Test paper I

1. (a) Discuss the principle of Mercator's projection. Why are Mercator's chart specifically suitable for Navigational purposes ?
   (b) A mercator's chart is constructed to a scale of 15cm = 1° of longitude. What is the length of the latitude scale between Latitudes 38° 40'N & 39° 50'N, on that chart.
2. An A.M. sight of the sun when worked with Lat 51° 55'N gave an observed longitude of 20° 04'W and when worked with Lat 52° 05'N gave a longitude of 19° 54½'N. What was the true bearing of the sun ?
3. Two ships A & B in latitude 35° S are 60° Longitude apart. Both ships now steam towards each other along a great circle track. If A's speed is twice the speed of 'B' how far would 'B' have moved when they meet ?
4. Define the following terms :
   (a) Meridional parts (b) Sidereal year (c) Quadrature of the moon (d) Retrograde motion of a planet.
5. Explain why a star crosses the meridian four minutes earlier each day ?
6. If the earth's axis was not titled at an angle of 23½° to a plane at right angles to its orbit, what effect would it have on the length of day and night & on the seasons on the earth's surface and why ?

Test paper II

1. Draw a figure of the celestial sphere on the plane on the rational horizon for an observer in lat 30°N (use radius = 4.5cm). Indicate in the figure the following :–
   (a) Position of Aries when its LHA = 50°
   (b) Position of star with Decl 15° S & SHA = 270°
   (c) Complete the PZX triangle.
   (d) Measure the approximate True altitude & azimuth of the body in your figure.

(e) State what projection you have used for the figure, and what each point, line & arc represent in your figure.

2. On 21st July 76, for a stationary observer in Long 10° 07.1'E when GHA Aries was 14° 05.5' a star was observed to be on his upper merdian with a true altitude of 89° 38' bearing south. Later when the same star was on his lower meridian it had a true altitude of 24° 21' bearing south. Find the latitude of the observer and identify the star.

3. Explain with a suitable diagram, why planet Venus sometimes appears as a morning star and sometimes as an evening star. Why is Venus not observeable close to midnight in tropical latitudes ?

4. Define the following terms :-
   (a) Ecliptic (b) Latitude of a place (c) Difference of Longitude
   (d) Synodic period of a clestial body.

5. What are 'v' & 'd' corrections tabulated in the nautical almanac How are they applied ? Why is 'v' correction sometimes negative for Venus ?

6. What is the International date line ? Explain & illustrate how and why the date changes while crossing the International date line.

Test paper III

1. Prove the following statements with appropriate figures
   (a) GHA Aries + SHA ★ - Long (W) = LHA ★
   (b) LMT - LAT = + Equation of time
   (c) LMT mer. pass. of Aries + RA ★ = LMT Mer. pass of ★

2. If sun's amplitude at Summer solstice at rising was E 29° N find its true altitude when on the prime vertical for the same stationary observer.

3. In fog a vessel anchors in DR. Lat 40° N. When fog cleared a sun sight is taken and when worked by Longitude by chronometer method, using 40°N lat, gave a certain longitude. At the same time, the position found by sextant angles of shore objects put the vessel 6 nautical miles further North and 5 nautical miles further East. Find the sun's true bearing.

4. Write short notes on the following :
   (a) Nautical Twilight
   (b) Augmentation to Moon's semidiameter.
   (c) Liberation in Latitude & Long of moon.
   (d) Maximum elongation of an inferior planet.

5. (a) What correction is applied to the wireless bearing of shore

... chart ? Why and how are they applied ?

(b) Explain the principle of a polar Gnomonic chart, How is such a chart used for the purpose of navigation ?

Test paper IV

1. (a) Given the following back observation of the Sun's L.L. find the true altitude and Zenith distance.
   Sextant altitude 101° 34', I.E. 2.5' on the arc, H.E. 11m.
   (b) When using an artificial horizon, show that the apparent altitude = Observed altitude ÷ 2
2. Why & how does equation of time become zero four times a year ?
3. To an observer in North latitude star Arcturus (Dec. 19° 25½' N) bore 000° (T) when on the meridian. Later its true altitude at maximum azimuth was 24° 29' Find the observer's Latitude.
4. Explain & illustrate how solar eclipse takes place ? What conditions are necessary to have solar eclipse ? why is there no solar eclipse every new moon day ? What is meant by path of an eclipse ?
5. Write short notes on the following :
   (a) Superior & Inferior conjunction.
   (b) International date line.
   (c) Vertex of a Great Circle track.
   (d) The nodes of the moon.
6. Find the difference in the speed of earth's rotation between two places on the earth in Lat 25° & 48°.

ANSWERS TO TEST PAPERS

Paper I

Q. 1 (b) Ans. 22.5 cm.
Q. 2 120½°(T)
Q. 3 967 Miles.

Paper II

Q. 1 (d) Approxi True Alt. 31°
AZ 134° (T) as measured.
(e) Equidistant projection.
2. Lat 57° (S) star Achnar.

Paper III

Q. 2 44° 10'
Q. 3 129 (T)

Paper IV

I (a) True Alt = 101° 41 96'
TZD = 11° 41.96'
(3) Lat 7° 55½N
(6) 213.46 m.p.h.